GENERAL COMMENTS

Grade 12 Pre-Calculus Mathematics Achievement Test (January 2018)

Student Performance—Observations

The following observations are based on local marking results and on comments made by markers during the sample marking session. These comments refer to common errors made by students at the provincial level and are not specific to school jurisdictions.

Information regarding how to interpret the provincial test and assessment results is provided in the document *Interpreting and Using Results from Provincial Tests and Assessments* available at [www.edu.gov.mb.ca/k12/assess/support/results/index.html.](www.edu.gov.mb.ca/k12/assess/support/results/index.html)

Various factors impact changes in performance over time: classroom-based, school-based, and home-based contexts, changes to demographics, and student choice of mathematics course. In addition, Grade 12 provincial tests may vary slightly in overall difficulty although every effort is made to minimize variation throughout the test development and pilot testing processes.

When considering performance relative to specific areas of course content, the level of difficulty of the content and its representation on the provincial test vary over time according to the type of test questions and learning outcomes addressed. Information regarding learning outcomes is provided in the document *Grades 9 to 12 Mathematics: Manitoba Curriculum Framework of Outcomes* (2014).

Unit A: Transformations of Functions (provincial mean: 72.0%)

Conceptual Knowledge

Most students were able to correctly state the domain of $f^{-1}(x)$ and $\frac{1}{f(x)}$, but some used reciprocal instead of inverse or inverse instead of reciprocal. When given a graph and asked to sketch the graph of its inverse, some students confused inverse with a vertical reflection. When asked to determine the equation of a function in terms of another function, most students were able to determine the vertical translation, but many struggled with vertical reflection. Some students made errors in the order of translations. A few students forgot to include the original function in their new equation. The horizontal translation was well done, but some students mistakenly included a horizontal reflection. When sketching composite functions, students did not understand that if one function had a restricted domain, then the resulting function should also be restricted. Many students confused composition of functions with multiplication of functions.

Procedural Skill

Students generally knew domain but did not know how to write the domain of a discrete function. When sketching the graph of $f^{-1}(x)$, some students had one incorrect point on the graph, commonly the point on the line $y = x$. When asked to describe the transformation, students did not look at the question as a transformation of a graph. Some students explained changing forms between the two, rather than talking about the graphs.

Communication

When graphing, many students forgot the arrowhead or switched the arrowhead and the endpoint. When stating the domain, many students made notation errors for inequality notation and incorrect brackets. When determining the value of composite functions, many students made notation errors when identifying their answers. Students had difficulties describing the transformations using appropriate vocabulary.

Unit B: Trigonometric Functions (provincial mean: 69.0%)

Conceptual Knowledge

In general, students were able to determine the values of specific angles. However, some students gave incorrect quadrants or incorrect values. Some students struggled with graphing a $y = \tan x$ graph. Students were able to determine the amplitude and vertical shift but often struggled with horizontal stretch of a sinusoidal function. When asked to determine if a point is on the unit circle, many students only included the special triangle values. When asked to explain how the horizontal stretch affects the period, many students did not consider the absolute value or the explanation was not clear.

Procedural Skill

Students knew most of the values of special angles but mixed up the quadrants or multiplied the values when they were supposed to add. They knew that $\csc\theta$ was the reciprocal of $\sin\theta$, but often gave the incorrect quadrant signs. Arithmetic errors were numerous. When graphing the tan function, many students gave incorrect asymptotes or only gave one period and left out scales. Students often confused the horizontal stretch with the period.

Communication

When writing trigonometric functions, students had notation errors such as writing $\sin$ instead of $\sin\theta$. Incorrect signs for quadrants were given quite often. Students appeared to forget to check in what quadrant the angle terminated. They often changed an equation to an expression, and did not use brackets correctly. When drawing a graph, many students forgot the scales on the axes. Many graphs were not very accurate and did not stay within the correct range once translated. Students did not always simplify their final answer or failed to state a final answer. Their answers to explain questions often had terminology errors or there was lack of clarity in the explanation.

Unit C: Binomial Theorem (provincial mean: 67.3%)

Conceptual Knowledge

In general, students struggled with determining the number of ways that people could not sit next to each other. They had an understanding of how to order the total number of people but struggled with the concept of grouping objects together. Most students were able to correctly use the fundamental counting principle to solve, when repetition was allowed, but many forgot to account for zero being an acceptable digit to use when creating a code. When asked to determine the power on a binomial given information about a specific term, most students were able to determine the correct value for k. However, most students had difficulty correctly substituting into the formula. Many students attempted to solve this question with a pattern and did not account for the powers on the terms. Overall, students were able to understand that a binomial with an odd power would have an even number of terms but some students incorrectly stated the binomial as having n-1 terms. When required to solve an equation involving a combination, some students were unsure which formula to use and how to use it correctly.

Procedural Skill

Students knew how to calculate permutations correctly, using either the formula or the fundamental counting principle. Students had difficulty using the binomial theorem formula to solve for a power and often substituted the coefficients when equating to the power of a given term. Some students made algebraic errors when simplifying using power laws to solve for an unknown. While many students understood that a binomial with an odd exponent would not have a middle term, they often did not reference an understanding of the number of terms in the expansion. When evaluating an equation involving combinations, many students struggled with expanding the factorials and doing the correct algebra to solve for the variable.

Communication

When determining the exponent on the binomial, some students changed an equation to an expression after substituting into the formula. When expanding factorials, some students made notation errors such as misplacing the factorial sign inside the brackets or forgetting the brackets altogether. When solving the equation involving combinations, many students made errors in the substitution or simplification of the formulas that led to impossible solutions (fractions or negative values) that were not rejected. Students also often changed the equation to an expression throughout their work in solving for n.

Unit D: Polynomial Functions (provincial mean: 72.7%)

Conceptual Knowledge

When asked to determine the zeros of a given polynomial function, most students were able to correctly use strategies to solve for the zeros. However, some students did not understand the concept of determining the zeros of a polynomial function and expressed their final answer as a product of factors. Most students did not show the use of the remainder theorem. Some students were unable to factor a quadratic expression where $a \neq 1$. Many students were unable to recognize the form of the division statement given in a question where they were asked to explain why $x-a$ wasn't a factor of a given polynomial function, $P(x)$, and then wrote the equation of $P(x)$. When asked to sketch the graph of a polynomial function, some students omitted to include a *y*-intercept and/or sketched a polynomial function with incorrect end behaviour. Some students plotted the *x*-intercepts with opposite signs. Other students included an extra *x*-intercept to accommodate for their inability to demonstrate a multiplicity of three on a given factor of the polynomial function and still arrived at the correct end behaviour.

Procedural Skill

Even though most students were able to use synthetic division correctly, some had difficulty with the procedures or forgot to use the correct sign for the divisor. Some students neglected to include their initial value as a factor of the polynomial function and therefore were unable to solve for all of the zeros of $P(x)$. When graphing a polynomial function, some students had trouble graphing the correct multiplicity of three, which resulted in graphs with incorrect shape. When asked to solve for the zeros of a polynomial function, some students did not equate the function to zero before solving the equation. Other students left the function in factored form without solving for the zeros.

Communication

When graphing polynomial functions, sometimes scales were not indicated on axes and/or arrowheads were omitted. When asked to solve for the zeros of a polynomial function, some students changed the equation into an expression. Students used poor terminology when explaining why a given expression, $x-a$, was not a factor of a given polynomial function, $P(x)$. Many students demonstrated lack of clarity in their explanations.

Unit E: Trigonometric Equations and Identities (provincial mean: 69.9%)

Conceptual Knowledge

Students generally knew the required steps to solve the trigonometric equation algebraically, but had difficulty executing their steps. Overall, students were able to substitute the correct reciprocal and double angle identities as required. Students had difficulty determining the correct values of the reciprocal identity. Most students were able to determine the value of θ_r, but had difficulty determining the value of the angle(s) θ in the appropriate quadrants. In solving the trigonometric equations, students had difficulty knowing when and how $(x \in \mathbb{R},$ rather than $x \in \mathbb{Z})$ to express the answer as a general solution. When proving trigonometric identities, some students had difficulty with the logical process.

Procedural Skills

Students had difficulty factoring when solving quadratic trigonometric equations. When proving trigonometric identities, some students had difficulty with the algebraic strategies, such as making a common denominator, splitting an angle into two, or cancelling trigonometric ratios.

Some students rejected $\csc\theta = 4$ before substituting the reciprocal identity. Also, students had difficulty with the special angle values.

Communication

Students often changed an equation to an expression when solving equations. They wrote an angle as a trigonometric ratio, and omitted or interchanged variables. In some cases, students did not simplify their final answer or stated the final answer in degrees instead of expressing the final answer as an equation.

Unit F: Exponents and Logarithms (provincial mean: 67.3%)

Conceptual Knowledge

When solving a logarithm word problem involving a comparison, some students did not substitute correctly into the given formula. Many students had difficulty manipulating the formula and changing the logarithms into exponential form. Some students correctly applied logarithms but had difficulty using the product law. When asked to describe how to find the solution to an exponential function equated to a radical function, some students simply stated the value of x instead of describing in words. Other students did not understand how to use the graphs of two functions to find a solution for x and believed they needed to find the intersection point. When using the laws of logarithms to expand a logarithmic expression, some students struggled with the product law and did not recognize that a coefficient and variable needed to be separated in order to completely expand. Many students were able to use the power law when the exponent in the argument was a whole number, but did not apply the power law when the argument was in radical form. Some left the argument in radical form; others changed the radical to a rational exponent but did not use the power law for complete expansion. When graphing an exponential function, many students did not recognize that a fractional base would result in a decreasing function and instead sketched the graph as an increasing exponential function. Some students did not recognize that a value subtracted from an exponent of x would result in a horizontal translation and instead applied a vertical translation. When asked to describe how transformations could be used obtain the graph of a logarithmic function from the graph of an exponential function, many students simply stated that the graphs were logarithmic or exponential, instead of describing their inverse relationship.

Procedural Skill

Some students were able to correctly use the formula in a logarithmic word problem to convert into exponential form but then struggled when using a comparison to find a solution. Several students had difficulty isolating x when applying logarithms to solve an exponential equation with uncommon bases using algebra. Algebraic strategies such as collecting like terms with x and isolating x to determine a quotient of logarithms were difficult for students. Some students did not recognize how to completely expand a logarithmic expression and instead tried to equate the expression to x for solution. When graphing an exponential function, some students did not plot the y-intercept, which resulted in an incorrect shape of the graph. Some students made arithmetic errors in their work when solving a logarithmic equation for an unknown base. Many students understood that the graph of a logarithmic function could be used to find the inverse graph of an exponential function but did not know how to describe the concept in words or lacked the correct terminology to do so. Some students did not describe how to transform the graphs and other students mistook inverse for reciprocal when describing the transformation.

Communication

When making a comparison with logarithmic word problems, some students did not recognize that a comparison requires an answer to three decimal places, and instead left their answers in exponential form. When applying the power law to solve for *x* in an exponential equation with uncommon bases, many students did not include brackets around the binomial power. Some students did not include arrowheads on the graph of an exponential function, especially when the graph was approaching a horizontal asymptote. Other students sketched the graph with the correct behaviour approaching the *x*-axis as *x* approaches infinity but did not sketch the horizontal asymptote. Some students drew the exponential function crossing over the horizontal asymptote.

Unit G: Radicals and Rationals (provincial mean: 61.3%)

Conceptual Knowledge

One outcome in this unit is to determine the solution of radical equations by analyzing a graph. When determining the solution by the *x*-value of the intersection of two lines, students stated the solution as the coordinate point, rather than just the *x*-value of the point. They failed to see that the original equation has no *y*-value, therefore, there should not be a *y*-value in the solution. When students were asked to describe how to apply the radical function to a point, they failed to address that the *x*-coordinate of the point would not be affected. Many seemed to understand the concept, but couldn’t describe it in words. When asked to graph the radical graph from an absolute value graph, some students thought that they needed to include asymptotes at the *x*-intercepts, and others scribbled out the negative *y*-values on the original graph to show that they would not exist on the radical graph. The rational graphs lacked points of discontinuity and had incorrect asymptotes, so the graphs took on a variety of shapes. When asked for the range of the graph, students would give the domain, or would miss the point of discontinuity.

Procedural Skill

On the rational graphs, students would show through their work that a point of discontinuity existed, but then would graph an asymptote instead. Some students did not know how to write a range where there were two values to be excluded. When asked to write the equation of the asymptotes, students would write that $x \neq \text{value}$ and $y \neq \text{value}$ rather than using the equals sign. Some used abbreviations such as HA or VA and did not give the equation. Other students did not know how to determine the vertical asymptote of a rational function when there was a binomial in the denominator. In stating the horizontal and vertical asymptotes, some students used $x = \text{value}$ for the horizontal asymptote and $y = \text{value}$ for the vertical asymptote. On the explain questions, some students would demonstrate that they knew how to solve the question, but were unable to explain the concept.

Communication

When asked to write the equation of a radical graph, some students included an f in their equation or failed to indicate that it was a radical. Some students labelled the point of discontinuity incorrectly. The graphing of the radical graph was quite sloppy, with the part of the graph between invariant points not clearly drawn. When writing the range, students had bracket errors and some incorrect inequality notations. On some of the "explain" questions, students often did not address what the question was asking, or forgot that *explain* and *describe* mean that they have to answer the question in words instead of justifying with mathematical solutions.

Communication Errors

Errors that are not related to the concepts or procedures are called "Communication Errors" and these were tracked on the *Answer/Scoring Sheet* in a separate section. There was a maximum ½ mark deduction for each type of communication error committed, regardless of the number of errors per type (i.e., committing a second error for any type did not further affect a student's mark).

The following table indicates the percentage of students who had at least one error for each type.

E1 final answer	▪ answer given as a complex fraction ▪ final answer not stated ▪ impossible solution(s) not rejected in final answer and/or in steps leading to final answer	17.9%
E2 equation/expression	▪ changing an equation to an expression or vice versa ▪ equating the two sides when proving an identity	35.7%
E3 variables	▪ variable omitted in an equation or identity ▪ variables introduced without being defined	13.8%
E4 brackets	▪ " $\sin x^2$ " written instead of " $\sin^2 x$ " ▪ missing brackets but still implied	12.7%
E5 units	▪ units of measure omitted in final answer ▪ incorrect units of measure ▪ answer stated in degrees instead of radians or vice versa	2.1%
E6 rounding	▪ rounding error ▪ rounding too early	12.1%
E7 notation/transcription	▪ notation error ▪ transcription error	48.6%
E8 domain/range	▪ answer outside the given domain ▪ bracket error made when stating domain or range ▪ domain or range written in incorrect order	7.7%
E9 graphing	▪ endpoints or arrowheads omitted or incorrect ▪ scale values on axes not indicated ▪ coordinate points labelled incorrectly	45.3%
E10 asymptotes	▪ asymptotes drawn as solid lines ▪ asymptotes omitted but still implied ▪ graph crosses or curls away from asymptotes	17.0%

Marking Accuracy and Consistency

Information regarding how to interpret the marking accuracy and consistency reports is provided in the document *Interpreting and Using Results from Provincial Tests and Assessments* available at [www.edu.gov.mb.ca/k12/assess/support/results/index.html.](http://www.edu.gov.mb.ca/k12/assess/support/results/index.html)

These reports compare the local marking results to the results from the departmental re-marking of sample test booklets. Provincially, 36.0% of the test booklets sampled resulted in a higher score locally than those given at the department; in 7.5% of the cases, local marking resulted in a lower score. Overall, the accuracy of local versus central marking for the test was consistent. To highlight this consistency, 56.5% of the booklets sampled and marked by the department received a central mark within $\pm 2\%$ of the local mark and 97.3% of the sampled booklets were within $\pm 6\%$. Scores awarded at the local level were, on average, 1.3% higher than the scores given at the department.

Survey Results

Teachers who supervised the Grade 12 Pre-Calculus Mathematics Achievement Test in January 2018 were invited to provide comments regarding the test and its administration. A total of 107 teachers responded to the survey. A summary of their comments is provided below.

After adjusting for non-responses:

- 100% of the teachers indicated that all of the topics in the test were taught by the time the test was written.
- 97.1% of the teachers indicated that the test content was consistent with the learning outcomes as outlined in the curriculum document. 99.0% of teachers indicated that the reading level of the test was appropriate and 98.0% of them thought the test questions were clear.
- 97.2% and 91.9% of the teachers, respectively, indicated that students were able to complete the questions requiring a calculator and the entire test in the allotted time.
- 98.1% of the teachers indicated that their students used a formula sheet throughout the semester and 100% of teachers indicated that their students used the formula sheet during the test.
- 39.4% of the teachers indicated that graphing calculators were incorporated during the instruction of the course and 96.2% of teachers indicated that the use of a scientific calculator was sufficient for the test.