Grade 8 Mathematics

Support Document for Teachers

Manitoba

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Available in alternate formats upon request.

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Grade 8 Mathematics Blackline Masters (BLMs)

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Introduction

Purpose of This Document

Grade 8 Mathematics: Support Document for Teachers provides various suggestions for instruction, assessment strategies, and learning resources that promote the meaningful engagement of mathematics learners in Grade 8. The document is intended to be used by teachers as they work with students in achieving the learning outcomes and achievement indicators identified in *Kindergarten to Grade 8 Mathematics: Manitoba Curriculum Framework of Outcomes (2013)* (Manitoba Education).

Background

Kindergarten to Grade 8 Mathematics: Manitoba Curriculum Framework of Outcomes is based on *The Common Curriculum Framework for K–9 Mathematics*, which resulted from ongoing collaboration with the Western and Northern Canadian Protocol (WNCP). In its work, WNCP emphasized

- common educational goals
- the ability to collaborate and achieve common goals
- high standards in education
- planning an array of educational activities
- removing obstacles to accessibility for individual learners
- optimum use of limited educational resources

The growing effects of technology and the need for technology-related skills have become more apparent in the last half century. Mathematics and problem-solving skills are becoming more valued as we move from an industrial to an informational society. As a result of this trend, mathematics literacy has become increasingly important. Making connections between mathematical study and daily life, business, industry, government, and environmental thinking is imperative. The Kindergarten to Grade 12 mathematics curriculum is designed to support and promote the understanding that mathematics is

- a way of learning about our world
- part of our daily lives
- both quantitative and geometric in nature

Beliefs about Students and Mathematics Learning

The Kindergarten to Grade 8 mathematics curriculum is designed with the understanding that students have unique interests, abilities, and needs. As a result, it is imperative to make connections to all students' prior knowledge, experiences, and backgrounds.

Students are curious, active learners with individual interests, abilities, and needs. They come to classrooms with unique knowledge, life experiences, and backgrounds. A key component in successfully developing numeracy is making connections to these backgrounds and experiences.

Students learn by attaching meaning to what they do, and they need to construct their own meaning of mathematics. This meaning is best developed when learners encounter mathematical experiences that proceed from the simple to the complex and from the concrete to the abstract. The use of manipulatives and a variety of pedagogical approaches can address the diversity of learning styles and developmental stages of students. At all levels, students benefit from working with a variety of materials, tools, and contexts when constructing meaning about new mathematical ideas. Meaningful student discussions can provide essential links among concrete, pictorial, and symbolic representations of mathematics.

Students need frequent opportunities to develop and reinforce their conceptual understanding, procedural thinking, and problem-solving abilities. By addressing these three interrelated components, students will strengthen their ability to apply mathematical learning to their daily lives.

The learning environment should value and respect all students' experiences and ways of thinking, so that learners are comfortable taking intellectual risks, asking questions, and posing conjectures. Students need to explore problem-solving situations in order to develop personal strategies and become mathematically literate. Learners must realize that it is acceptable to solve problems in different ways and that solutions may vary.

Conceptual understanding: comprehending mathematical concepts, relations, and operations to build new knowledge. (Kilpatrick, Swafford, and Findell 5)

Procedural thinking: carrying out procedures flexibly, accurately, efficiently, and appropriately.

Problem solving: engaging in understanding and resolving problem situations where a method or solution is not immediately obvious. (OECD 12)

First Nations, Métis, and Inuit Perspectives

First Nations, Métis, and Inuit students in Manitoba come from diverse geographic areas with varied cultural and linguistic backgrounds. Students attend schools in a variety of settings, including urban, rural, and isolated communities. Teachers need to recognize and understand the diversity of cultures within schools and the diverse experiences of students.

First Nations, Métis, and Inuit students often have a whole-world view of the environment; as a result, many of these students live and learn best in a holistic way. This means that students look for connections in learning, and learn mathematics best when it is contextualized and not taught as discrete content.

Many First Nations, Métis, and Inuit students come from cultural environments where learning takes place through active participation. Traditionally, little emphasis was placed upon the written word. Oral communication along with practical applications and experiences are important to student learning and understanding.

A variety of teaching and assessment strategies are required to build upon the diverse knowledge, cultures, communication styles, skills, attitudes, experiences, and learning styles of students. The strategies used must go beyond the incidental inclusion of topics and objects unique to a culture or region, and strive to achieve higher levels of multicultural education (Banks and Banks).

Affective Domain

A positive attitude is an important aspect of the affective domain that has a profound effect on learning. Environments that create a sense of belonging, encourage risk taking, and provide opportunities for success help students develop and maintain positive attitudes and self-confidence. Students with positive attitudes toward learning mathematics are likely to be motivated and prepared to learn, participate willingly in classroom learning activities, persist in challenging situations, and engage in reflective practices.

Teachers, students, and parents* need to recognize the relationship between the affective and cognitive domains, and attempt to nurture those aspects of the affective domain that contribute to positive attitudes. To experience success, students must be taught to set achievable goals and assess themselves as they work toward reaching these goals.

Striving toward success and becoming autonomous and responsible learners are ongoing, reflective processes that involve revisiting the setting and assessment of personal goals.

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* In this document, the term *parents* refers to both parents and guardians and is used with the recognition that in some cases only one parent may be involved in a child's education.

Middle Years Education

Middle Years education is defined as the education provided for young adolescents in Grades 5, 6, 7, and 8. Middle Years learners are in a period of rapid physical, emotional, social, moral, and cognitive development.

Socialization is very important to Middle Years students, and collaborative learning, positive role models, approval of significant adults in their lives, and a sense of community and belonging greatly enhance adolescents' engagement in learning and commitment to school. It is important to provide students with an engaging and social environment within which to explore mathematics and to construct meaning.

Adolescence is a time of rapid brain development when concrete thinking progresses to abstract thinking. Although higher-order thinking and problem-solving abilities develop during the Middle Years, concrete, exploratory, and experiential learning is most engaging to adolescents.

Middle Years students seek to establish their independence and are most engaged when their learning experiences provide them with a voice and choice. Personal goal setting, co-construction of assessment criteria, and participation in assessment, evaluation, and reporting help adolescents take ownership of their learning. Clear, descriptive, and timely feedback can provide important information to the mathematics student. Asking open-ended questions, accepting multiple solutions, and having students develop personal strategies will help students to develop their mathematical independence.

Adolescents who see the connections between themselves and their learning, and between the learning inside the classroom and life outside the classroom, are more motivated and engaged in their learning than those who do not observe these connections.

Adolescents thrive on challenges in their learning, but their sensitivity at this age makes them prone to discouragement if the challenges seem unattainable. Differentiated instruction allows teachers to tailor learning challenges to adolescents' individual needs, strengths, and interests. It is important to focus instruction on where students are and to see every contribution as valuable.

The energy, enthusiasm, and unfolding potential of young adolescents provide both challenges and rewards to educators. Those educators who have a sense of humour and who see the wonderful potential and possibilities of each young adolescent will find teaching in the Middle Years exciting and fulfilling.

Mathematics Education Goals for Students

The main goals of mathematics education are to prepare students to

- communicate and reason mathematically
- use mathematics confidently, accurately, and efficiently to solve problems
- appreciate and value mathematics
- make connections between mathematical knowledge and skills and their applications
- commit themselves to lifelong learning
- become mathematically literate citizens, using mathematics to contribute to society and to think critically about the world

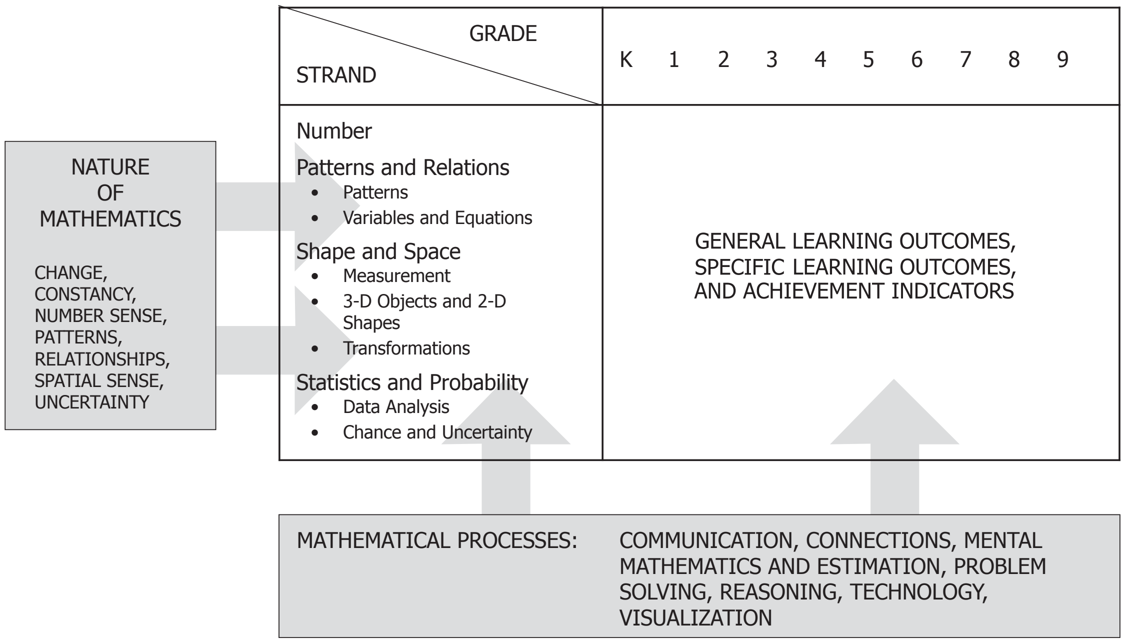
> Mathematics education must prepare students to use mathematics to think critically about the world.

Students who have met these goals will

- gain understanding and appreciation of the contributions of mathematics as a science, a philosophy, and an art
- exhibit a positive attitude toward mathematics
- engage and persevere in mathematical tasks and projects
- contribute to mathematical discussions
- take risks in performing mathematical tasks
- exhibit curiosity

Conceptual Framework for Kindergarten to Grade 9 Mathematics

The chart below provides an overview of how mathematical processes and the nature of mathematics influence learning outcomes.



Mathematical Processes

There are critical components that students must encounter in mathematics to achieve the goals of mathematics education and encourage lifelong learning in mathematics.

Students are expected to

- communicate in order to learn and express their understanding
- connect mathematical ideas to other concepts in mathematics, to everyday experiences, and to other disciplines
- demonstrate fluency with mental mathematics and estimation
- develop and apply new mathematical knowledge through problem solving
- develop mathematical reasoning
- select and use technologies as tools for learning and solving problems
- develop visualization skills to assist in processing information, making connections, and solving problems

The common curriculum framework incorporates these seven interrelated mathematical processes, which are intended to permeate teaching and learning:

- **Communication [C]:** Students communicate daily (orally, through diagrams and pictures, and by writing) about their mathematics learning. They need opportunities to read about, represent, view, write about, listen to, and discuss mathematical ideas. This enables them to reflect, to validate, and to clarify their thinking. Journals and learning logs can be used as a record of student interpretations of mathematical meanings and ideas.
- **Connections [CN]:** Mathematics should be viewed as an integrated whole, rather than as the study of separate strands or units. Connections must also be made between and among the different representational modes—concrete, pictorial, and symbolic (the symbolic mode consists of oral and written word symbols as well as mathematical symbols). The process of making connections, in turn, facilitates learning. Concepts and skills should also be connected to everyday situations and other curricular areas.
- **Mental Mathematics and Estimation [ME]:** The skill of estimation requires a sound knowledge of mental mathematics. Both are necessary to many everyday experiences, and students should be provided with frequent opportunities to practise these skills. Mental mathematics and estimation is a combination of cognitive strategies that enhances flexible thinking and number sense.
- **Problem Solving [PS]:** Students are exposed to a wide variety of problems in all areas of mathematics. They explore a variety of methods for solving and verifying problems. In addition, they are challenged to find multiple solutions for problems and to create their own problems.
- **Reasoning [R]:** Mathematics reasoning involves informal thinking, conjecturing, and validating—these help students understand that mathematics makes sense. Students are encouraged to justify, in a variety of ways, their solutions, thinking processes, and hypotheses. In fact, good reasoning is as important as finding correct answers.
- **Technology [T]:** The use of calculators is recommended to enhance problem solving, to encourage discovery of number patterns, and to reinforce conceptual development and numerical relationships. They do not, however, replace the development of number concepts and skills. Carefully chosen computer software can provide interesting problem-solving situations and applications.
- **Visualization [V]:** Mental images help students to develop concepts and to understand procedures. Students clarify their understanding of mathematical ideas through images and explanations.

These processes are outlined in detail in *Kindergarten to Grade 8 Mathematics: Manitoba Curriculum Framework of Outcomes (2013)*.

Strands

The learning outcomes in the Manitoba curriculum framework are organized into four strands across Kindergarten to Grade 9. Some strands are further subdivided into substrands. There is one general learning outcome per substrand across Kindergarten to Grade 9.

The strands and substrands, including the general learning outcome for each, follow.

Number

- Develop number sense.

Patterns and Relations

- Patterns
  - Use patterns to describe the world and solve problems.
- Variables and Equations
  - Represent algebraic expressions in multiple ways.

Shape and Space

- Measurement
  - Use direct and indirect measure to solve problems.
- 3-D Objects and 2-D Shapes
  - Describe the characteristics of 3-D objects and 2-D shapes, and analyze the relationships among them.
- Transformations
  - Describe and analyze position and motion of objects and shapes.

Statistics and Probability

- Data Analysis
  - Collect, display, and analyze data to solve problems.
- Chance and Uncertainty
  - Use experimental or theoretical probabilities to represent and solve problems involving uncertainty.

Learning Outcomes and Achievement Indicators

The Manitoba curriculum framework is stated in terms of general learning outcomes, specific learning outcomes, and achievement indicators:

- **General learning outcomes** are overarching statements about what students are expected to learn in each strand/substrand. The general learning outcome for each strand/substrand is the same throughout the grades from Kindergarten to Grade 9.
- **Specific learning outcomes** are statements that identify the specific skills, understanding, and knowledge students are required to attain by the end of a given grade.
- **Achievement indicators** are samples of how students may demonstrate their achievement of the goals of a specific learning outcome. The range of samples provided is meant to reflect the depth, breadth, and expectations of the specific learning outcome. While they provide some examples of student achievement, they are not meant to reflect the sole indicators of success.

In this document, the word *including* indicates that any ensuing items **must be addressed** to meet the learning outcome fully. The phrase *such as* indicates that the ensuing items are provided for illustrative purposes or clarification, and are **not requirements that must be addressed** to meet the learning outcome fully.

Summary

The conceptual framework for Kindergarten to Grade 9 mathematics describes the nature of mathematics, the mathematical processes, and the mathematical concepts to be addressed in Kindergarten to Grade 9 mathematics. The components are not meant to stand alone. Learning activities that take place in the mathematics classroom should stem from a problem-solving approach, be based on mathematical processes, and lead students to an understanding of the nature of mathematics through specific knowledge, skills, and attitudes among and between strands. *Grade 8 Mathematics: Support Document for Teachers* is meant to support teachers to create meaningful learning activities that focus on formative assessment and student engagement.

Assessment

Authentic assessment and feedback are a driving force for the suggestions for assessment in this document. The purposes of the suggested assessment activities and strategies are to parallel those found in *Rethinking Classroom Assessment with Purpose in Mind: Assessment for Learning, Assessment as Learning, Assessment of Learning* (Manitoba Education, Citizenship and Youth). These include the following:

- assessing *for*, *as*, and *of* learning
- enhancing student learning
- assessing students effectively, efficiently, and fairly
- providing educators with a starting point for reflection, deliberation, discussion, and learning

Assessment *for* learning is designed to give teachers information to modify and differentiate teaching and learning activities. It acknowledges that individual students learn in idiosyncratic ways, but it also recognizes that there are predictable patterns and pathways that many students follow. It requires careful design on the part of teachers so that they use the resulting information to determine not only what students know, but also to gain insights into how, when, and whether students apply what they know. Teachers can also use this information to streamline and target instruction and resources, and to provide feedback to students to help them advance their learning.

Assessment *as* learning is a process of developing and supporting metacognition for students. It focuses on the role of the student as the critical connector between assessment and learning. When students are active, engaged, and critical assessors, they make sense of information, relate it to prior knowledge, and use it for new learning. This is the regulatory process in metacognition. It occurs when students monitor their own learning and use the feedback from this monitoring to make adjustments, adaptations, and even major changes in what they understand. It requires that teachers help students develop, practise, and become comfortable with reflection, and with a critical analysis of their own learning.

Assessment *of* learning is summative in nature and is used to confirm what students know and can do, to demonstrate whether they have achieved the curriculum learning outcomes, and, occasionally, to show how they are placed in relation to others. Teachers concentrate on ensuring that they have used assessment to provide accurate and sound statements of students' proficiency so that the recipients of the information can use the information to make reasonable and defensible decisions.

Overview of Planning Assessment			
	Assessment *for* Learning	**Assessment *as* Learning**	**Assessment *of* Learning**
Why Assess?	■ to enable teachers to determine next steps in advancing student learning	■ to guide and provide opportunities for each student to monitor and critically reflect on his or her learning and identify next steps	■ to certify or inform parents or others of the student's proficiency in relation to curriculum learning outcomes
Assess What?	■ each student's progress and learning needs in relation to the curriculum outcomes	■ each student's thinking about his or her learning, what strategies he or she uses to support or challenge that learning, and the mechanisms he or she uses to adjust and advance his or her learning	■ the extent to which each student can apply the key concepts, knowledge, skills, and attitudes related to the curriculum outcomes
What Methods?	■ a range of methods in different modes that make a student's skills and understanding visible	■ a range of methods in different modes that elicit the student's learning and metacognitive processes	■ a range of methods in different modes that assess both product and process
Ensuring Quality	■ accuracy and consistency of observations and interpretations of student learning ■ clear, detailed learning expectations ■ accurate, detailed notes for descriptive feedback to each student	■ accuracy and consistency of a student's self-reflection, self-monitoring, and self-adjustment ■ engagement of the student in considering and challenging his or her thinking ■ the student records his or her own learning	■ accuracy, consistency, and fairness of judgments based on high-quality information ■ clear, detailed learning expectations ■ fair and accurate summative reporting
Using the Information	■ provide each student with accurate descriptive feedback to further his or her learning ■ differentiate instruction by continually checking where each student is in relation to the curriculum outcomes ■ provide parents or guardians with descriptive feedback about student learning and ideas for support	■ provide each student with accurate, descriptive feedback that will help him or her develop independent learning habits ■ have each student focus on the task and his or her learning (not on getting the right answer) ■ provide each student with ideas for adjusting, rethinking, and articulating his or her learning ■ provide the conditions for the teacher and student to discuss alternatives ■ the student reports about his or her own learning	■ indicate each student's level of learning ■ provide the foundation for discussions on placement or promotion ■ report fair, accurate, and detailed information that can be used to decide the next steps in a student's learning

Source: Manitoba Education, Citizenship and Youth. *Rethinking Classroom Assessment with Purpose in Mind: Assessment for Learning, Assessment as Learning, Assessment of Learning*. Winnipeg, MB: Manitoba Education, Citizenship and Youth, 2006, 85.

INSTRUCTIONAL FOCUS

The Manitoba curriculum framework is arranged into four strands. These strands are not intended to be discrete units of instruction. The integration of learning outcomes across strands makes mathematical experiences meaningful. Students should make the connection between concepts both within and across strands.

Consider the following when planning for instruction:

- Routinely incorporating conceptual understanding, procedural thinking, and problem solving within instructional design will enable students to master the mathematical skills and concepts of the curriculum.
- Integration of the mathematical processes within each strand is expected.
- Problem solving, conceptual understanding, reasoning, making connections, and procedural thinking are vital to increasing mathematical fluency, and must be integrated throughout the program.
- Concepts should be introduced using manipulatives and gradually developed from the concrete to the pictorial to the symbolic.
- Students in Manitoba bring a diversity of learning styles and cultural backgrounds to the classroom and they may be at varying developmental stages. Methods of instruction should be based on the learning styles and abilities of the students.
- Use educational resources by adapting to the context, experiences, and interests of students.
- Collaborate with teachers at other grade levels to ensure the continuity of learning of all students.
- Familiarize yourself with exemplary practices supported by pedagogical research in continuous professional learning.
- Provide students with several opportunities to communicate mathematical concepts and to discuss them in their own words.

"Students in a mathematics class typically demonstrate diversity in the ways they learn best. It is important, therefore, that students have opportunities to learn in a variety of ways—individually, cooperatively, independently, with teacher direction, through hands-on experience, through examples followed by practice. In addition, mathematics requires students to learn concepts and procedures, acquire skills, and learn and apply mathematical processes. These different areas of learning may involve different teaching and learning strategies. It is assumed, therefore, that the strategies teachers employ will vary according to both the object of the learning and the needs of the students" (Ontario 24).

DOCUMENT ORGANIZATION AND FORMAT

This document consists of the following sections:

- **Introduction:** The Introduction provides information on the purpose and development of this document, discusses characteristics of and goals for Middle Years learners, and addresses Aboriginal perspectives. It also gives an overview of the following:
  - **Conceptual Framework for Kindergarten to Grade 9 Mathematics:** This framework provides an overview of how mathematical processes and the nature of mathematics influence learning outcomes.
  - **Assessment:** This section provides an overview of planning for assessment in mathematics, including assessment *for*, *as*, and *of* learning.
  - **Instructional Focus:** This discussion focuses on the need to integrate mathematics learning outcomes and processes across the four strands to make learning experiences meaningful for students.
  - **Document Organization and Format:** This overview outlines the main sections of the document and explains the various components that comprise the various sections.
- **Number:** This section corresponds to and supports the Number strand for Grade 8 from *Kindergarten to Grade 8 Mathematics: Manitoba Curriculum Framework of Outcomes (2013)*.
- **Patterns and Relations:** This section corresponds to and supports the Patterns and Variables and Equations substrands of the Patterns and Relations strand for Grade 8 from *Kindergarten to Grade 8 Mathematics: Manitoba Curriculum Framework of Outcomes (2013)*.
- **Shape and Space:** This section corresponds to and supports the Measurement, 3-D Objects and 2-D Shapes, and Transformations substrands of the Shape and Space strand for Grade 8 from *Kindergarten to Grade 8 Mathematics: Manitoba Curriculum Framework of Outcomes (2013)*.
- **Statistics and Probability:** This section corresponds to and supports the Data Analysis and Chance and Uncertainty substrands of the Statistics and Probability strand for Grade 8 from *Kindergarten to Grade 8 Mathematics: Manitoba Curriculum Framework of Outcomes (2013)*.
- **Blackline Masters (BLMs):** Blackline masters are provided to support student learning. They are available in *Microsoft Word* format so that teachers can alter them to meet students' needs, as well as in *Adobe* PDF format.
- **Bibliography:** The bibliography lists the sources consulted and cited in the development of this document.

Guide to Components and Icons

Each of the sections supporting the strands of the Grade 8 Mathematics curriculum includes the components and icons described below.

Enduring Understanding(s):

These statements summarize the core idea of the particular learning outcome(s). Each statement provides a conceptual foundation for the learning outcome. It can be used as a pivotal starting point in integrating other mathematics learning outcomes or other subject concepts. The integration of concepts, skills, and strands remains of utmost importance.

General Learning Outcome(s):

General learning outcomes (GLOs) are overarching statements about what students are expected to learn in each strand/substrand. The GLO for each strand/substrand is the same throughout Kindergarten to Grade 8.

Specific Learning Outcome(s):	**Achievement Indicators:**
Specific learning outcome (SLO) statements define what students are expected to achieve by the end of the grade. A code is used to identify each SLO by grade and strand, as shown in the following example: **8.N.1** The first number refers to the grade (Grade 8). The letter(s) refer to the strand (Number). The last number indicates the SLO number. [C, CN, ME, PS, R, T, V] Each SLO is followed by a list indicating the applicable mathematical processes.	Achievement indicators are examples of a representative list of the depth, breadth, and expectations for the learning outcome. The indicators may be used to determine whether students understand the particular learning outcome. These achievement indicators will be addressed through the learning activities that follow.

Prior Knowledge

Prior knowledge is identified to give teachers a reference to what students may have experienced previously.

Related Knowledge

Related knowledge is identified to indicate the connections among the Grade 8 Mathematics learning outcomes.

Background Information

Background information is provided to give teachers knowledge about specific concepts and skills related to the particular learning outcome(s).

Mathematical Language

Lists of terms students will encounter while achieving particular learning outcomes are provided. These terms can be placed on mathematics word walls or used in a classroom mathematics dictionary. *Kindergarten to Grade 8 Mathematics Glossary: Support Document for Teachers* (Manitoba Education, Citizenship and Youth) provides teachers with an understanding of key terms found in Kindergarten to Grade 8 mathematics. The glossary is available on the Manitoba Education and Advanced Learning website at <www.edu.gov.mb.ca/k12/cur/math/supports.html>.

Learning Experiences

Suggested instructional strategies and assessment ideas are provided for the specific learning outcomes and achievement indicators. In general, learning activities and teaching strategies related to specific learning outcomes are developed individually, except in cases where it seems more logical to develop two or more learning outcomes together. Suggestions for assessment include information that can be used to assess students' progress in their understanding of a particular learning outcome or learning experience.

Assessing Prior Knowledge

Suggestions are provided to assess students' prior knowledge and to help direct instruction.

Observation Checklist

Checklists are provided for observing students' responses during lessons.

Suggestions for Instruction

The instructional suggestions include the following:

- **Achievement indicators appropriate to particular learning experiences are listed.**

- **Materials/Resources:** Outlines the resources required for a learning activity.
- **Organization:** Suggests groupings (individual, pairs, small group, and/or whole class).
- **Procedure:** Outlines detailed steps for implementing suggestions for instruction.

Some learning activities make use of BLMs, which are found in the Blackline Masters section in *Microsoft Word* and *Adobe* PDF formats.

Putting the Pieces Together

Putting the Pieces Together tasks, found at the end of the learning outcomes, consist of a variety of assessment strategies. They may assess one or more learning outcomes across one or more strands and may make cross-curricular connections.

Grade 8 Mathematics

Number

Number and Shape and Space (Measurement)—8.N.1, 8.N.2, 8.SS.1

Enduring Understandings:

The square roots of perfect squares are rational numbers.

The square roots of non-perfect squares are irrational numbers.

Many geometric properties and attributes of shapes are related to measurement.

General Learning Outcomes:

Develop number sense.

Use direct or indirect measurement to solve problems.

SPECIFIC LEARNING OUTCOME(S):	ACHIEVEMENT INDICATORS:
8.N.1 Demonstrate an understanding of perfect squares and square roots, concretely, pictorially, and symbolically (limited to whole numbers). [C, CN, R,V]	➛ Represent a perfect square as a square region using materials, such as grid paper or square shapes. ➛ Determine the factors of a perfect square, and explain why one of the factors is the square root and the others are not. ➛ Determine whether or not a number is a perfect square using materials and strategies such as square shapes, grid paper, or prime factorization, and explain the reasoning. ➛ Determine the square root of a perfect square, and record it symbolically. ➛ Determine the square of a number.
8.N.2 Determine the approximate square root of numbers that are not perfect squares (limited to whole numbers). [C, CN, ME, R, T]	➛ Estimate the square root of a number that is not a perfect square using the roots of perfect squares as benchmarks. ➛ Approximate the square root of a number that is not a perfect square using technology (e.g., calculator, computer).

continued

Specific Learning Outcome(s):	**Achievement Indicators:**
	➛ Explain why the square root of a number shown on a calculator may be an approximation. ➛ Identify a number with a square root that is between two given numbers.
8.SS.1 Develop and apply the Pythagorean theorem to solve problems. [CN, PS, R, T, V]	➛ Model and explain the Pythagorean theorem concretely, pictorially, or by using technology. ➛ Explain, using examples, that the Pythagorean theorem applies only to right triangles. ➛ Determine whether or not a triangle is a right triangle by applying the Pythagorean theorem. ➛ Solve a problem that involves determining the measure of the third side of a right triangle, given the measures of the other two sides. ➛ Solve a problem that involves Pythagorean triples (e.g., 3, 4, 5 or 5, 12, 13).

Prior Knowledge

Students may have had experience with the following:

- Demonstrating an understanding of regular and irregular 2-D shapes by
  - recognizing that area is measured in square units
  - selecting and justifying referents for the units cm^2 or m^2
  - estimating area by using referents for cm^2 or m^2
  - determining and recording area (cm^2 or m^2)
  - constructing different rectangles for a given area (cm^2 or m^2) in order to demonstrate that many different rectangles may have the same area
- Solving problems involving 2-D shapes and 3-D objects
- Designing and constructing different rectangles given either perimeter or area, or both (whole numbers), and drawing conclusions
- Identifying and sorting quadrilaterals, including
  - rectangles

- squares
- trapezoids
- parallelograms
- rhombuses

according to their attributes

- Developing and applying a formula for determining the
  - perimeter of polygons
  - area of rectangles
  - volume of right rectangular prisms
- Constructing and comparing triangles, including
  - scalene
  - isosceles
  - equilateral
  - right
  - obtuse
  - acute

  in different orientations

Background Information

Squares and Square Roots

A *square* is a 2-dimensional (2-D) shape with all four sides equal.

The total area the square covers is measured in square units.

To determine the side length of a square when given the area, the square root must be determined.

A *perfect square* can be described as

- a square with whole number sides (e.g., 1 × 1, 2 × 2, 3 × 3)
- a number whose square root is an integer (e.g., $\sqrt{4}$ = 2 or –2)

A *non-perfect square* can be described as

- a square with non-whole number sides (e.g., 1.2 × 1.2)
- a number whose square root is not a whole number (e.g., $\sqrt{2}$)

Rounding is often used to determine the approximate square root of non-perfect squares.

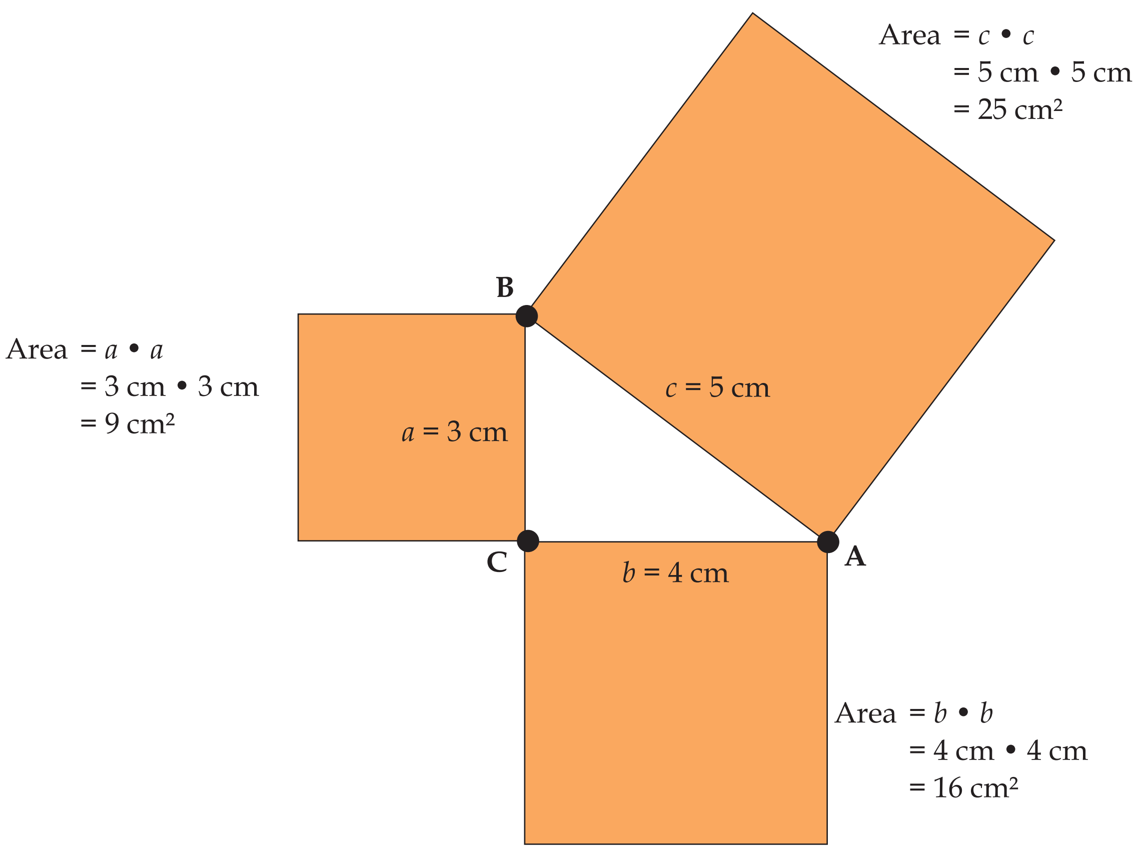
Pythagorean Theorem

The *Pythagorean theorem* states that, in a right triangle,

- the sum of the squares of the legs is equal to the square of the length of the hypotenuse
- the sum of the area of the squares formed on the legs is equal to the area of the square formed on the hypotenuse
- if the lengths of the legs are a and b and the hypotenuse is c, then $a^2 + b^2 = c^2$

Example:

We can determine whether triangle ABC, shown below, is a right triangle by checking whether the Pythagorean relationship is present.



If $a^2 + b^2 = c^2$, then triangle ABC is a right angle triangle:

$$9\text{ cm}^2 + 16\text{ cm}^2 \stackrel{?}{=} 25\text{ cm}^2$$

$$25\text{ cm}^2 = 25\text{ cm}^2 \checkmark$$

Therefore, triangle ABC is a right angle triangle.

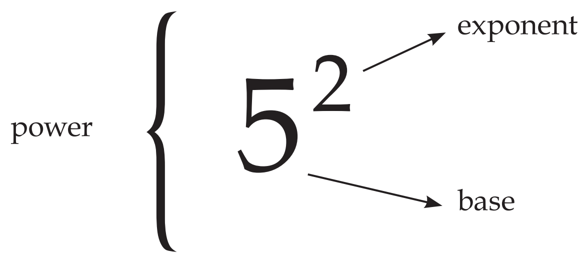
If you are given two of the values (a, b, or c) in a right angle triangle, you can determine the missing value by solving the equation.

Powers

This is the first formal experience that students will have with powers. A simple introduction will be needed to explain the role of the exponent and the base.

- *Power*: A short-hand, symbolic representation of repeated multiplication (e.g., $5^2 = 5 \bullet 5$).
- *Base*: The factor in a power; what is being repeatedly multiplied (e.g., in 5^2, 5 is the base).
- *Exponent*: The number in a power that tells how many factors there are; the number of factors in a repeated multiplication (e.g., in 5^2, 2 is the exponent).

Example:



Mathematical Language

factors

hypotenuse

perfect square

prime factorization

prime numbers

Pythagorean relationship (Pythagorean theorem)

right triangles

square root

Assessing Prior Knowledge

Materials: Grid paper

Organization: Individual

Procedure:

1. Tell students that they will be learning about the Pythagorean theorem over the next few lessons; however, you first need to determine what they already know.
2. Provide students with grid paper and have them draw a square with the side length 5 cm.
3. Have students determine the area of the square they just drew. Ask them how they found the area (e.g., Did they count the squares? Did they multiply length times width?).
4. Ask students to determine the side length of a square that has an area of 64 cm^2. They may use the grid paper if they need it to determine the length.
5. Have students draw a right triangle with one leg 4 cm and one leg 3 cm.

Observation Checklist

☑ Observe students' responses to determine whether they can do the following:

- ❒ Determine the area of a square.
- ❒ Find the side length of a perfect square.
- ❒ Understand the property that determines right triangles (90° angle).
- ❒ Draw right triangles.

Suggestions for Instruction

- **Represent a perfect square as a square region, using materials such as grid paper or square shapes.**
- **Determine the square of a number.**

Materials: BLM 5–8.9: Centimetre Grid Paper, straightedge, math journals, BLM 8.N.1.1: Determining Squares, calculators

Organization: Individual/whole class

Procedure:

1. Tell students that by the end of this lesson they will understand squares and square roots of perfect squares.
2. Each student needs a piece of graph paper (or see BLM 5–8.9: Centimetre Grid Paper).
3. Ask students to explain the difference between a square and a rectangle. (A *square* is a special kind of rectangle in that all four sides are equal.)
4. Ask students to draw a 2 × 2 square and label the length and width of the square 2 × 2. Then have them count the units within the boundary of the drawn square (the area) and write that amount inside the square.
5. Ask students to repeat the above, making a 3 × 3 square, a 4 × 4 square, and a 5 × 5 square.
6. Ask students to explain, in their math journals, any connections they have noticed between the length and width of the square and the area of the square. (Explanations can include words, pictures, diagrams, symbols, and so on.) The hope is that students will notice that the area is determined by multiplying the length by the width and that the two factors are always the same (e.g., 2 × 2, 3 × 3, 4 × 4, 5 × 5).
7. Ask students whether they have ever seen another way to write 2 × 2, 3 × 3, 4 × 4, 5 × 5, and so on. This question will generate discussion on saying 2 squared or 2^2, and so on.
8. Have a brief class discussion about powers, bases, and exponents to ensure students understand the process of, and the symbolic notation for, squaring a number.
9. Provide students with copies of BLM 8.N.1.1: Determining Squares and ask them to determine the squares of a variety of numbers with or without the use of a calculator.

Observation Checklist

☑ Observe students' responses to determine whether they can do the following:

- ❒ Determine the square of a number, given a picture.
- ❒ Determine the square of a number symbolically.
- ❒ Use mental mathematics strategies and number sense to determine the square of a number.

Suggestions for Instruction

- **Determine the square root of a perfect square, and record it symbolically.**

Materials: Square tiles, calculators, graph paper, math journals,
BLM 8.N.1.2: Determining Square Roots

Organization: Pairs/whole class/individual

Procedure:

1. Give students square tiles, and allow them a few minutes to explore the manipulatives.
2. Ask students to make squares with varying side lengths. Have them record the side lengths and areas of the squares in a table.
3. Record class data to form a table of side lengths and areas for squares of lengths 1 to 15.
4. Ask students to add to the table a third column titled Area as Power, and have them record the areas of the first 15 squares as powers.

   Example:

Side Length (units)	Area (units^2)	Area as Power
1	1	1^2
2	4	2^2

5. Ask students whether they have ever heard the term *square root*. Generate discussion, using questions such as the following:
   - What is the relationship between the area of a square and the length of its side?
   - We just learned how to square a number. What do you think *square root* could be?
   - What does the word *root* mean?
6. As a class review, remind students that when they find the square of a number, they are finding the area of a square, which can be written as 2^2, and so on.
7. Determining the square root of a number is determining what number was multiplied by itself to get the square.

   Examples:
   - If the square of a number is 4, what number multiplied by itself is 4? Answer: 2
   - If the square of a number is 16, what number multiplied by itself is 16? Answer: 4

   Since the square of a number is actually the area of the square, the square root of a number is the same as finding the length and/or width of the square.
8. Ask students to add to the table a fourth column titled Side Length as Square Root, and have them record the side lengths of the first 15 squares as square roots.

   Example:

Side Length (units)	Area (units^2)	Area as Power	Side Length as Square Root
1	1	1^2	$\sqrt{1}$
2	4	2^2	$\sqrt{4}$

9. Have students brainstorm, in pairs, what patterns they see in the table. Discuss these patterns as a class.
10. Demonstrate to students the square root button on a calculator so that they can find the square roots of numbers. If they do not have a square root button, brainstorm ideas about how they can use a calculator to determine the square root (guess and check).
11. Have students individually complete BLM 8.N.1.2: Determining Square Roots.
12. Have students insert a piece of graph paper into their math journals, and ask them to represent 81 as a square region on the grid. Check whether they have drawn a 9×9 square. Ask them to identify the square and square root of the square region.

Observation Checklist

☑ Observe students' responses to determine whether they can do the following:

- ❒ Determine the square root of a number, given a model.
- ❒ Determine the square root of a number symbolically.
- ❒ Use mental mathematics strategies and number sense to determine the square root of a number.

Suggestions for Instruction

- **Determine the factors of a perfect square, and explain why one of the factors is the square root and the others are not.**
- **Determine whether or not a number is a perfect square using materials and strategies such as square shapes, grid paper, or prime factorization, and explain the reasoning.**
- **Determine the square root of a perfect square and record it symbolically.**
- **Estimate the square root of a number that is not a perfect square using the roots of perfect squares as benchmarks.**
- **Approximate the square root of a number that is not a perfect square using technology (e.g., calculator, computer).**

Materials: Square paper cut into individual squares (BLM 5–8.6: Blank Hundred Squares or BLM 5–8.9: Centimetre Grid Paper), math journals, calculator or computer

Organization: Individual/pairs/whole class

Procedure:

Part A

1. Give each pair of students a handful of square paper tiles. (Make sure some groups end up with an amount that forms a perfect square and some end up with an amount that does not.)
2. Have students make a square using all the tiles they received. (Students may need to be encouraged to use parts of squares. For example, if a group is given 10 tiles, students would use nine whole tiles and divide the tenth tile to create a square whose sides are slightly larger than three tiles.)
3. Make a class chart stating the area of the square and the length of the side.
4. Have students make observations of what is happening
5. Take pictures of students' squares or have students tape these down for use in Part F.

Observation Checklist

☑ Observe students' responses to determine whether they can do the following:

- ❒ Understand that the space covered by a shape is its area.
- ❒ Recognize that a square is a quadrilateral with four equal sides that meet at right angles.
- ❒ Make the connection between having no squares left over and having a perfect square.

Part B

1. Tell students that they will learn two more ways, other than using a calculator, to determine the square root of a perfect square.
2. List all the factors of 36 (a perfect square). When students are listing the factors, observe whether they notice a way that will help them to find the square root.

   Examples:

   - If students list the factors forming a rectangle, it is easier for them to identify the square root.

     1, 36

     2, 18

     3, 12

     4, 9

     6

     6 is listed only once because it is multiplied by itself to get 36.

     6 is the square root of 36.

     OR

   - If students list the factors in order from lowest to highest, the square root is always the median number.

     1, 2, 3, 4, (6,) 9, 12, 18, 36

3. Have students list all the factors of a few more perfect squares to identify the square root of those squares.

Observation Checklist

☑ Observe students' responses to determine whether they can do the following:

- ❒ Determine the factors of a number.
- ❒ Use factors to determine the square root of a number.
- ❒ Apply mental mathematics and number sense strategies to determine the square root of a number.

Part C

1. Ask students to write down the following numbers and categorize them as perfect squares or non-perfect squares: 16, 25, 36, 27, 32, 42.
2. Review with students what a *prime number* is and what a *factor* is. Demonstrate the prime factorization of several numbers.
3. Model, using 16 as an example, how to write numbers as the product of prime factors (2 • 2 • 2 • 2). Have students write the next five numbers as the product of prime factors.
4. After ensuring that all students have the correct response, use a Think-Pair-Share strategy by having students individually examine the prime factorization to determine whether there is any pattern or rule that can be determined, and then having them share it with a partner. Elicit answers from the class to form a general rule. (Perfect squares have an even number of prime factors.)

   Examples:

   - Written as a product of prime numbers, 36 is 2 • 2 • 3 • 3. There are two 2s and two 3s, so 36 must be a perfect square.
   - Written as a product of prime numbers, 27 is 3 • 3 • 3. There are three 3s (an uneven number of 3s in the product of prime numbers), so 27 must be a non-perfect square.

Observation Checklist

☑ Observe students' responses to determine whether they can do the following:

- ❒ Determine the prime factors of a number.
- ❒ Develop an understanding of using prime factors to determine the square root of a number.
- ❒ Apply mental mathematics and number sense strategies to determine the square root of a number.

Part D

1. Using the number 36 from the previous learning activity, have students determine the square root of a perfect square. Write 36 as a product of prime factors on the whiteboard.
2. Have students divide the factors 2 • 2 • 3 • 3 into two equal groups (2 • 3 and 2 • 3). Ask them to multiply the two groups and see what number they end up with.
3. Ask students to write 144 as a product of prime factors.
4. Have students separate the prime factors into two equal groups and multiply. Record their results to determine the square root of 144.
5. Ask students whether they can determine the square root of numbers in another way.
6. Discuss students' responses to generate other ways to determine the square root of perfect squares.

Observation Checklist

☑ Observe students' responses to determine whether they can do the following:

- ❒ Determine the square root of perfect squares using prime factorization.
- ❒ Develop a personal strategy for determining square roots.

Part E

1. Have students select one of the following and complete a math journal entry with words, pictures, tables, symbols, and so on.
   - Explain how prime factorization (or factoring) can be used to determine whether a number is a perfect square. Provide examples to help support your response.
   - Select a number that is a perfect square and a number that is not. Select a method for proving that the number is or is not a perfect square. Describe why you chose the method you did.

Observation Checklist

☑ Observe students' responses to determine whether they can do the following:

- ❐ Determine the square root of perfect squares.
- ❐ Use materials, diagrams, and symbols to determine the square root of a number.
- ❐ Develop a personal strategy for determining square roots.

Part F

1. Using the squares created in Part A, have students work in pairs to discuss the difference between squares made from a perfect square number of paper tiles and those made from a number of tiles that is not a perfect square (e.g., 25 squares vs. 30 squares).
2. Facilitate a class discussion about the square roots of numbers that are not perfect squares.
3. Have students work in pairs to estimate the square roots of each group's square and then check responses using technology.
4. Discuss as a class the strategies that students used to determine the square roots of numbers that are not perfect squares.

Observation Checklist

☑ Observe students' responses to determine whether they can do the following:

- ❐ Use number sense to approximate the square root of numbers that are not perfect squares.
- ❐ Reason mathematically.

Suggestions for Instruction

- **Estimate the square root of a number that is not a perfect square using the roots of perfect squares as benchmarks.**
- **Approximate the square root of a number that is not a perfect square using technology (e.g., calculator, computer).**
- **Explain why the square root of a number shown on a calculator may be an approximation.**
- **Identify a number with a square root that is between two given numbers.**

Materials: Math journals

Organization: Small group/whole class

Procedure:

1. Tell students that they will be learning how to identify and explain the approximate square root of non-perfect squares.
2. Draw a number line and write the numbers 25 to 36 above it.
3. Underneath the line, have students write the square root of 25 and 36 directly under the numbers. (They are setting the benchmarks for the next step.)
4. Working in small groups, students estimate the square root of 30 and explain their thinking. Have a reporter from each group present to the class the group's decision. It should fall between 5 and 6 (around 5.5) since 30 is approximately midway between 25 and 36.
5. In their groups, students estimate the square root of 54 using benchmarks and justify their responses.
6. As students are working in their groups, listen to the dialogue to observe the level of understanding students have. Record your observations.
7. You may need to repeat this process a few more times with non-perfect squares. Then ask students questions such as the following:
   - Why are the numbers you have been finding the square roots of non-perfect squares?
   - Why do we find the approximate square roots of them?
   - Can you determine an exact value with your calculator? The numbers are non-repeating, non-terminating. Calculators will round the number based on the space available on the screen.
8. Ask students to respond to the following in their math journals:
   - Based on your understanding of perfect and non-perfect squares and their square roots, identify a number that will have a square root between 4 and 5.
   - Use diagrams, tables, materials, symbols, words, and/or numbers to justify your choice.

Observation Checklist

☑ Observe students' responses to determine whether they can do the following:

- ❒ Use number sense to approximate the square root of a non-perfect square.
- ❒ Explain why the square root of a non-perfect square may be an approximation.
- ❒ Reason mathematically.

- **Identify a number with a square root that is between two given numbers.**
- **Determine the square root of a perfect square, and record it symbolically.**
- **Determine the square of a number.**
- **Estimate the square root of a number that is not a perfect square using the roots of perfect squares as benchmarks.**

Materials: BLM 8.N.1.3: I Have . . . , Who Has . . . ?

Organization: Whole class/small group

Procedure:

1. Tell students that they will be playing a square root version of the game, I Have . . . , Who Has . . . ? (see BLM 8.N.1.3). Explain that each student will get one card (some students will need to have more cards if there are fewer than 30 students in the class). One student will start the game by reading his or her card, and the person who has the answer to the question posed by the student reads his or her card. Play continues in this fashion until it gets back to the person who started the game.
2. After students have played the game several times, have them make their own square root game and play it with the other members of the class.

Variation: Have students work in groups of two or three, giving them several cards. Play the game as described in the procedure above. This variation gives students the opportunity to engage with more than one card.

Observation Checklist

☑ Observe students' responses to determine whether they can do the following:

- ❒ Determine the square of a number.
- ❒ Determine the square root of a perfect square.
- ❒ Determine the square root of a non-perfect square.

Suggestions for Instruction

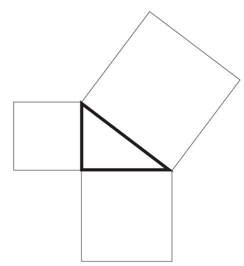
- **Model and explain the Pythagorean theorem concretely, pictorially, or by using technology.**
- **Explain, using examples, that the Pythagorean theorem applies only to right triangles.**
- **Determine whether or not a triangle is a right triangle by applying the Pythagorean theorem.**
- **Solve a problem that involves Pythagorean triples (e.g., 3, 4, 5 or 5, 12, 13).**

Materials: Two colours of 1 cm grid paper (BLM 5–8.9: Centimetre Grid Paper), ruler, sharp pencil, scissors

Organization: Small group

Procedure:

1. Tell students that they are going to determine a unique relationship between the legs of a right triangle and the hypotenuse of the right triangle. They will be using their understanding of squares and square roots.
2. Draw a right triangle and label the triangle so that students understand what the legs of the triangle are and what the hypotenuse is.
3. Have students form small groups, and provide them with the following instructions:



   - Using one colour of grid paper, draw a right triangle with one leg 3 cm and one leg 4 cm.
   - Using the other colour of grid paper, cut out squares large enough to fit along the edge of each of the legs and the hypotenuse.
   - Count the area of each square, combining partial squares to make whole squares.
   - Describe what relationship you see between the areas of the squares.
   - Try the procedure again to see if your theory is correct. This time, draw a right triangle with one leg 6 cm and the other leg 8 cm.
   - Try it one more time, this time with one leg 9 units and the other leg 12 units. (At this point, the small groups should have noticed that the sum of the areas of the squares off the legs is equal to the area of the square off the hypotenuse.)
4. Work with students to help them express their thoughts using mathematical language and symbols. Lead them to the generalization that $a^2 + b^2 = c^2$.
5. Have students draw other non-right angle triangles (e.g., 4 cm, 5 cm, 8 cm or 4 cm, 6 cm, 7 cm). Discuss with students what they notice. Lead them to the generalization that the Pythagorean theorem applies only to right triangles.

Observation Checklist

☑ Observe students' responses to determine whether they can do the following:

- ❐ Explain the Pythagorean theorem using pictures.
- ❐ Demonstrate that the Pythagorean theorem applies only to right triangles.

Suggestions for Instruction

- **Solve a problem that involves determining the measure of the third side of a right triangle, given the measures of the other two sides.**

Materials: Two colours of 1 cm grid paper (BLM 5–8.9: Centimetre Grid Paper), calculator, ruler, scissors

Organization: Pairs/whole class

Procedure:

1. Provide students with several triangles from which the length of the hypotenuse is missing.
2. Have students work in pairs to develop a procedure for determining the length of that missing side.
3. Discuss various procedures as a class.
4. If the symbolic procedure does not come up in the discussion, demonstrate this procedure for the class and allow students to practise determining the measure of the hypotenuse.
5. Repeat steps 1 to 4 for triangles from which the length of one of the legs is missing.

Observation Checklist

☑ Observe students' responses to determine whether they can do the following:

- ❒ Apply prior knowledge of squares and square roots.
- ❒ Develop a procedure for determining the measure of a missing side in a right angle triangle.
- ❒ Apply a procedure for determining the measure of a missing side in a right angle triangle.

Suggestions for Instruction

- **Determine whether or not a triangle is a right triangle by applying the Pythagorean theorem.**
- **Solve a problem that involves determining the measure of the third side of a right triangle, given the measures of the other two sides.**

Materials: BLM 8.N.1.4: Pythagorean Theorem, calculator

Organization: Individual/pairs

Procedure:

1. Provide students with a copy of BLM 8.N.1.4: Pythagorean Theorem.
2. Ask students to complete the BLM. Have them check with a learning partner if they experience difficulty.

Observation Checklist

☑ Observe students' responses to determine whether they can do the following:

- ❒ Determine the measure of a missing side in a right angle triangle.
- ❒ Apply the Pythagorean theorem to determine whether a triangle is a right triangle.

Pythagorean Theorem in Real Life

Introduction:

Students will have the opportunity to apply the Pythagorean theorem to everyday situations by solving everyday problems.

Purpose:

Students should have a good understanding of the Pythagorean theorem before being assigned this task. They will need to know how to create equivalent proportions to create a scale model of the scenarios provided below. In addition, they will need to know how to convert metres to centimetres.

Curricular Links: Art, English Language Arts (ELA)

Materials/Resources: Various art supplies

Organization: Individual or small group

If this is an individual learning activity, students can choose one of the following scenarios. Choice allows for adaptations and individual interests.

If this is a group learning activity, each member is responsible for one of the scenarios; however, students can receive assistance from their group members.

Inquiry:

Students will create a scaled version of one of the following scenarios. They must make the selected scenario appear realistic. They will create the appropriate length of the hypotenuse of the chosen scenario to the nearest tenth. They must complete a one-page explanation of all their mathematical work, including an explanation of how they created their scaled version and how they used the Pythagorean theorem to determine the lengths/distances.

Scenarios:

1. Fire fighters are called to an apartment fire. A family is trapped on the second floor. Fire fighters need to rescue the family using their extension ladder. The second-floor apartment is 3 metres from the ground. There are shrubs and a sidewalk jutting out 2 metres from the building. How long must the extension ladder be to reach the second-floor apartment?
2. Joey is trying out as a catcher for the local baseball team. Part of his evaluation involves showing how quickly he can throw the ball to second base. How far does he need to throw the ball if the bases are 90 feet apart?
3. Sam is starting his first day at a new job at a moving company. The truck box he is driving is 2 metres high. The end of the ramp is on the ground 4 metres from the back of the truck. How long is Sam's ramp?

Assessment:

The following rubric can be used to assess achievement of the mathematics learning outcomes.

Pythagorean Theorem in Real Life—Assessment				
Criteria	Meeting Expectations	Developing to Meet Expectations	Beginning to Meet Expectations	Incomplete
The student				
■ applies the Pythagorean theorem to solve problems	❒ provides a clear explanation of how the Pythagorean theorem was used to determine the length/distance	❒ provides a general explanation of how the Pythagorean theorem was used to determine the length/distance	❒ provides a vague or minimal explanation of how the Pythagorean theorem was used to determine length/distance	❒ provides no explanation of how the Pythagorean theorem was used to determine length/ distance
■ solves problems that involve proportions	❒ provides a clear explanation of how he or she determined the scale model for the scenario	❒ provides a general explanation of how he or she determined the scale model for the scenario	❒ provides a vague or minimal explanation of how he or she determined the scale model for the scenario	❒ provides no explanation of how he or she determined the scale model for the scenario

Extension:

Students could research other ways in which the Pythagorean theorem is used in everyday life and prepare a presentation on what they have learned. They could present their findings in the form of a demonstration, PowerPoint presentation, video, and so on.

NOTES

Number—8.N.3

Enduring Understandings:

Percents can be thought of as a ratio comparing to 100 or a fraction out of 100.

Percents can range from 0 to higher than 100.

Percents, fractions, decimals, and ratios are different representations of the same quantity.

Percents have the same value as their fraction, decimal, and ratio equivalent, and this can be useful in solving problems with percents.

General Learning Outcome:

Develop number sense.

Specific Learning Outcome(s):	**Achievement Indicators:**
8.N.3 Demonstrate an understanding of percents greater than or equal to 0%. [CN, PS, R, V]	➛ Provide a context where a percent may be more than 100% or between 0% and 1%. ➛ Represent a fractional percent using grid paper. ➛ Represent a percent greater than 100% using grid paper. ➛ Determine the percent represented by a shaded region on a grid, and record it in decimal, fractional, or percent form. ➛ Express a percent in decimal or fractional form. ➛ Express a decimal in percent or fractional form. ➛ Express a fraction in decimal or percent form. ➛ Solve a problem involving percents. ➛ Solve a problem involving combined percents (e.g., addition of percents, such as GST + PST). ➛ Solve a problem that involves finding the percent of a percent (e.g., A population increased by 10% one year and then increased by 15% the next year. Explain why there was not a 25% increase in population over the two years.).

Prior Knowledge

Students may have had experience with the following:

- Demonstrating an understanding of fractions by using concrete and pictorial representations to
  - create sets of equivalent fractions
  - compare fractions with like and unlike denominators
- Describing and representing decimals (tenths, hundredths, thousandths) concretely, pictorially, and symbolically
- Relating decimals to fractions (tenths, hundredths, thousandths)
- Comparing and ordering decimals (tenths, hundredths, thousandths) by using
  - benchmarks
  - place value
  - equivalent decimals
- Demonstrating an understanding of percent (limited to whole numbers) concretely, pictorially, and symbolically
- Solving problems involving percents from 1% to 100%
- Demonstrating an understanding of the relationship between repeating decimals and fractions, and terminating decimals and fractions

Background Information

People regularly encounter practical situations requiring them to understand and solve problems related to percent. These situations include problems related to sports statistics, price discounts, price increases, taxes, polls, social changes and trends, and the likelihood of precipitation. The media provide sources of contextual data for problems involving percent.

Learning outcome 8.N.3 builds on understandings related to fractions, ratios, decimals, percents, and problem solving that students developed in previous grades.

Fractions and Decimals

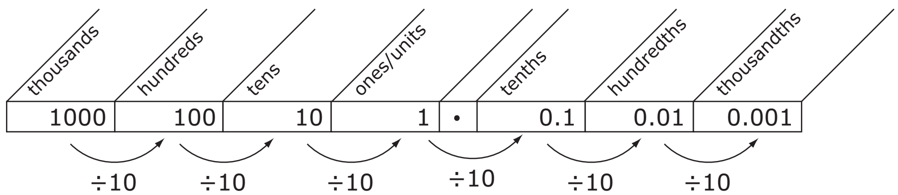
Before students become skilful at solving problems involving percent, they must have a strong conceptual understanding of fractions and decimals.

The term *fraction* has several meanings. An expert blends and separates these meanings for convenience, but this blending can confuse students who lack fluency in applying the different meanings of fraction. *Fraction notation* is used to represent a "cut" or part of a unit, a part of a group or set, a measure or point on a number line, a ratio, and a portion of a turn, and to indicate the division operation.

Decimals are a convenient way to represent fractional quantities using a place value system. Fractions may be converted to decimals by dividing the numerator by the denominator, or by finding an equivalent fraction with a denominator of 100.

A *decimal point* separates whole units from parts of units. Each position to the right of the decimal represents a tenth part of one of the previous units. The first position following the decimal represents a tenth part of one whole unit, and the second place represents a tenth part of a tenth or a hundredth part of one unit.

Example:



Problems Involving Percent

When translating standard notation to percent, the decimal point indicates where to read the hundredths in a number. The word *percent* means per hundred and may be substituted for the word *hundredths* when reading a number. Therefore, $\frac{7}{100}$ or 0.07 may be read as 7 hundredths and also as 7 percent.

Percent may also be used to represent fractional quantities that are a little larger than a hundredth. Each successive place value position represents one of the previous units cut into 10. For example, the third position represents a tenth of a hundredth part of a unit, or a thousandth part of one unit.

Our understanding of place value allows us to express any number as a number of selected units. Just as 141 can represent 14 tens and 1 one, 0.141 represents a number that is a little larger than one tenth of one whole. It may be expressed as 1.41 tenths, 14.1 hundredths, or 141 thousandths. When substituting the word *percent* as another word for hundredth, the decimal number 0.141 may be read as 14.1 hundredths, or 14.1 percent (%).

In Grade 8, students need to work with numbers from 0% to 1%, from 1% to 100%, and percents greater than 100%, including all fractional percents (e.g., $\frac{1}{4}$ %, $35\frac{1}{2}$ %, and $225\frac{3}{4}$ %).

With these various understandings of percent, students have multiple approaches to solving problems that involve percent:

- To find 25% of 80, students may think of the equivalent fraction $\frac{1}{4}$, and then find $\frac{1}{4}$ of 80.

  80 divided by 4 is 20, so 25% of 80 is 20.

- To find 0.1% of 200, students may use their understanding of place value to determine the value.

  100% of 200 is 200.

  10% of 200 is 200 ÷ 10, which is 20.

  1% of 200 is 200 ÷ 100, which is 2.0.

  0.1% of 200 is 200 ÷ 1000, which is 0.2.
- To find 120% of 40, students may use the knowledge that 120% is equivalent to 1.20, and so 120% of 40 is the same as 1.20 • 40, or 48.

Choosing numbers that are easy to work with will enable students to concentrate on the processes involved rather than on the arithmetic.

Where possible, use mental mathematics and the distributive property to find percents:

- Think of 35% as 25% + 10%. In the first problem above, 25% of 80 is 20, 10% of 80 is 8, and 20 + 8 = 28, so 35% of 80 is 28.
- To extend the second problem above, show how the distributive property is used for fractional percents. Think of how to determine 0.2% of 200. Think of 0.2% as 0.1% + 0.1%. Since 0.1% of 200 is 0.2, 0.2% of 200 would be 0.2 + 0.2 = 0.4.
- For the third problem above, think of 120% as 100% + 20%. 100% of 40 is 40 and 20% of 40 is 8 (the double of 10% of 40).

Students need to have a strong conceptual understanding of percent. Always start with hands-on activities to provide opportunities for students to develop that conceptual understanding. When converting fractional percents and percents greater than 100%, start with what students should already know—how to convert whole number percents less than 100% from percent to decimal form. Then, students can apply those same skills to converting fractional percents and percents greater than 100% to decimals and fractions. If students convert from percents to decimals first, they can then write the decimal as a fraction and simplify.

Example:

> To change a percent to a decimal, you divide by 100 (since percent means out of 100). This is true of fractional percents. For example, $85\frac{1}{2}$ % can be written in decimal form by thinking of it as a decimal percent (85.5%) and then showing this in decimal representation (85.5% ÷ 100% = 0.855).
>
> **Note:** Students often struggle with fractional percents because they see a fraction or decimal (e.g., $15\frac{1}{2}$ % or 22.75%) and already think the percent is a fraction or decimal. To clear up misconceptions, ask students whether they see the percent (%) sign. If yes, the number is still in percent form.

When combining percents, use various methods to solve the problems. It is important for students to be aware of the various methods but also know that one way is not better than the other. There may be fewer steps using one method over the other, but, depending on the learning style of the student, a longer method may be necessary. The ultimate goal is efficiency, which means the student is able to get accurate answers consistently and productively, using methods that the student understands.

Mathematical Language

combined percent
decimal
fraction
fractional percent
percent
GST
PST

Learning Experiences

Assessing Prior Knowledge

Materials: BLM 8.N.3.1: Percent Pre-Assessment, BLM 8.N.3.2: Percent Self-Assessment

Organization: Individual

Procedure:

1. Tell students that you need to find out what they already know about percents and that they are expected to do the best they can. Hand out BLM 8.N.3.1: Percent Pre-Assessment.
2. Have students work individually to complete the pre-assessment.
3. Assess students' work and provide descriptive feedback (rather than a mark) for the work they completed. Return each student's work.
4. Have students complete the Before Instruction column of BLM 8.N.3.2: Percent Self-Assessment prior to any instruction on percents.
5. At the end of the unit, have students complete the After Instruction column of BLM 8.N.3.2.

Note:

- This assessment is not to be used for marks but to obtain a benchmark of what students already understand about percents. This will help guide you to make instructional decisions and plan for students' individual needs.
- Have students complete BLM 8.N.3.1: Percent Pre-Assessment throughout the sequence of learning experiences related to percents. It is important for students to be aware of their learning progress.

Observation Checklist

☑ Observe students' responses to determine whether they can do the following:

- ❒ Represent fractional percents between 1% and 100% on a hundreds grid.
- ❒ Convert fractions to decimals and percents.
- ❒ Convert decimals to fractions and percents.
- ❒ Convert percents to fractions and decimals.
- ❒ Solve problems involving percents.

Learning Experiences

Suggestions for Instruction

- **Represent a fractional percent using grid paper.**
- **Represent a percent greater than 100% using grid paper.**
- **Determine the percent represented by a shaded region on a grid, and record it in decimal, fractional, or percent form.**

Materials: Graph paper, hundred grids (BLM 5–8.6: Blank Hundred Squares) or BLM 5–8.10: Base-Ten Grid Paper, BLM 8.N.3.3: Percent Grids

Organization: Individual

Procedure:

1. Tell students that by the end of this lesson they will be able to represent percents on graph paper/hundred grids and record percents from graph paper/hundred grids.
2. Review with students the definition of percent. *Percent* means out of each hundred.
3. Ask students the following questions:
   - How many squares are on the 10 × 10 graph paper or on the hundred grid? (100 squares)
   - How many squares would need to be shaded to represent 100%? (Shade in all squares.)
   - What is the value of one shaded square? (1%)
   - How would you represent 43%? (Shade in 43 squares.)

- How would you represent 74%? (Shade in 74 squares.)
- How would you represent $23\frac{1}{2}$%? (Shade in 23 full squares and $\frac{1}{2}$ of the 24th square.)
- How would you represent 150%? (Shade in one full grid and 50 squares on a second grid.)
- What does 150% mean to you? (Various answers—greater than 1)
- How would you represent $\frac{1}{2}$%? (Shade in half of one square.)
- How would you represent $\frac{3}{4}$%? ($\frac{3}{4}$ is less than one percent, so use only one square of a hundred grid. Divide it into four equal parts. Shade in three squares.)
- How would you represent 0.125%? (0.125% is the same as $\frac{1}{8}$%. $\frac{1}{8}$% is less than 1%, so use only one square of a hundred grid. Divide it into eight equal parts. Shade in one square.)

4. Provide students with graph paper or hundred grids. Ask them to represent the following fractions on their grids: 45%, 230%, $17\frac{2}{3}$%, and 0.2%.
5. Show each of the percent grids from BLM 8.N.3.3: Percent Grids, one at a time. Have students write the percent represented by each grid on an individual whiteboard and ask them to show you their responses.

Observation Checklist

☑ Observe students’ responses to determine whether they can do the following:

- ❒ Represent percents between 0% and 1%.
- ❒ Represent fractional percents less than 100%.
- ❒ Represent percents greater than 100% on graph paper or grids.
- ❒ Determine percents between 0% and 1%.
- ❒ Determine fractional percents less than 100%.
- ❒ Determine percents greater than 100% on graph paper or grids.

- **Provide a context where a percent may be more than 100% or between 0% and 1%.**

Materials: BLM 8.N.3.4: Percent Scenarios

Organization: Small group/whole class

Procedure:

1. Have students form small groups, and provide each group with a copy of BLM 8.N.3.4: Percent Scenarios.
2. Ask students to explain what the scenario statements mean and give reasons for their explanations. One presenter from each group then presents the group's explanation to the class.
3. Record ideas on a whiteboard or an overhead and work toward reaching consensus about the meaning of students' explanations.
4. Ask students to discuss fractional percents and percents greater than 100% with members of their household. Ask them to come to class the next day prepared to share a new real-world example from each category.
5. Have students share their examples, and see whether they are consistent with the meanings that students discussed in the previous class.

Observation Checklist

☑ Observe students' responses to determine whether they can do the following:

- ❒ Describe a scenario to represent percents between 0% and 1%.
- ❒ Describe a scenario to represent fractional percents greater than 100%.

Suggestions for Instruction

- **Express a percent in decimal or fractional form.**
- **Express a decimal in percent or fractional form.**
- **Express a fraction in decimal or percent form.**

Materials: Small whiteboards, margarine lids (or any other tool that students can use to record their answers and then quickly wipe off their responses in preparation for a new problem)

Organization: Whole class

Procedure:

1. Tell students that in this lesson they will learn how to represent fractions, decimals, and percents when the percents are fractional percents and when the percents are greater than 100%. They will use what they know from Grade 7 Mathematics (representing fractions, decimals, and percents) and extend it to new percents.
2. Ask students to write 75% on their whiteboard. Then ask them to write 75% as a decimal and a fraction.
3. Ask students what they did to go from the percent to the decimal.
4. Ask students what they did to go from the percent to the fraction.
5. Continue with whole-number percents ranging from 1% to 100% until the class has a solid foundation for how to convert percents to decimals and fractions. The procedures they describe here could be written on the board for reference.
6. Ask students to write 125% on their whiteboard. Then ask them to write 125% as a fraction and a decimal and explain how they did it.
7. Ask students whether they think their answer is reasonable (1.25 and $1\frac{1}{4}$ based on their understanding of percents and decimals).
8. Ask students to write $45\frac{1}{2}$% on their whiteboard. Then ask them to write $45\frac{1}{2}$% as a fraction and a decimal and explain how they did it.

   Note: For percents that contain fractional parts, students may experience some difficulty, as they may believe that the percent is already a fraction. It may help to get students to express these as a decimal percent (45.5%).
9. Ask students whether they think their answer is reasonable.
10. Ask students to write $\frac{3}{4}$% on their whiteboard. Then ask them to write $\frac{3}{4}$% as a fraction and a decimal and explain how they did it.
11. Ask students whether they think their answer is reasonable. (1% would be $\frac{4}{400}$. $\frac{3}{4}$% is a little smaller than 1%, and $\frac{3}{400}$ is just a little less than $\frac{4}{400}$, so the answer is reasonable.)

12. Continue to provide students with similar questions until they are demonstrating an understanding of the concept.
13. Repeat the above steps
    - with decimals (converting to fractions and percents)
    - with fractions (converting to decimals and percents)

Observation Checklist

☑ Observe students' responses to determine whether they can do the following:
- ❒ Convert fractions to decimals and percents.
- ❒ Convert decimals to fractions and percents.
- ❒ Convert percents to decimals and fractions.
- ❒ Use mathematical reasoning to determine the reasonableness of their answers.

Suggestions for Instruction

- **Solve a problem involving percents.**

Materials: Chart paper, markers, BLM 8.N.3.5: Percent Savings

Organization: Small group/whole class

Procedure:

1. Tell students that in this lesson they will learn how to solve problems involving percents.
2. Arrange students in groups of three or four, and assign one problem from BLM 8.N.3.5: Percent Savings to each group.
3. Students work together to solve the problem, showing their work on chart paper.
4. Each group presents the solution to their problem to the class. The problems are all similar, so, by the end, the class should have 10 exemplars that can be posted in the classroom for determining the percent saving.
5. Have students make up their own problems involving percents. They may switch questions with other groups and solve them using the solution(s) discussed in class.

Observation Checklist

☑ Observe students' responses to determine whether they can do the following:

- ❒ Apply problem-solving skills to determine an appropriate method to solve a problem with percents.
- ❒ Communicate mathematically within their group and with the class.
- ❒ Solve a problem involving percents.

Suggestions for Instruction

- **Solve a problem involving combined percents (e.g., addition of percent, such as GST + PST).**

Materials: Chart paper, markers, BLM 8.N.3.6: Final Cost

Organization: Small group/whole class

Procedure:

1. Arrange students in groups of three or four again (these groupings may be the same as or different from those of the previous learning experience). Assign each group one of the problems from BLM 8.N.3.6: Final Cost. Students must work together to solve the problem, showing their work on chart paper.
2. Each group presents the solution to their assigned problem to the class. The problems are all similar, so, by the end, the class should have 10 exemplars that can be posted in the classroom for determining how to combine percents.

   Note: If all the groups solve the problem the same way, they will have to show other methods of solving problems for combining percents. For example, you can calculate the percents separately, combine the tax percents first (PST + GST = 13%, then add that to the cost of an item), or combine the cost and tax percents (100% represents the cost of an item, 8% for PST and 5% of GST = 100% + 8% + 5% = 113%).

Observation Checklist

☑ Observe students' responses to determine whether they can do the following:

- ❒ Apply problem-solving skills to determine an appropriate method to solve a problem with combined percents.
- ❒ Communicate mathematically within their group and with the class.
- ❒ Solve a problem involving combined percents.

Suggestions for Instruction

- **Solve a problem that involves finding the percent of a percent (e.g., A population increased by 10% one year and then increased by 15% the next year. Explain why this was not a 25% increase in population over the two years.).**

Materials: Chart paper, markers, BLM 8.N.3.7: Percent Increase and Decrease

Organization: Small group/whole class

Procedure:

1. Tell students that they will determine whether a percent decrease (or increase) one time, and then an additional percent decrease (or increase) a second time, is the same thing as a total percent decrease (or increase).
2. Arrange students in groups of three or four. Give each group one of the two problems presented in BLM 8.N.3.7: Percent Increase and Decrease. Students work together to solve the problem, showing their work on chart paper.
3. Each group presents the solution to their problem to the class. Encourage student discussion and questions during this time. Add other scenarios as necessary.

Observation Checklist

☑ Observe students' responses to determine whether they can do the following:

- ❒ Apply problem-solving skills to determine an appropriate method to solve a problem with a percent of a percent.
- ❒ Communicate mathematically within their group and with the class.
- ❒ Solve a problem involving a percent of a percent.

Percents: My Understanding

Introduction:

Students are used to shopping for various items. Provide students with three items with the regular and sale prices included. They will need to determine the percent savings based on the prices, represent those percent savings on graph paper, and determine the total cost when PST and GST are included.

Purpose:

Students will represent percents on graph paper and show how they combine percents to find a total cost. This learning task should provide an assessment of many of the achievement indicators for learning outcome 8.N.3.

Curricular Links: ELA, Literacy with Information and Communication Technology (LwICT)

Materials/Resources: Access to flyers (or use the examples provided), technology (if students are representing their learning using ICT), poster paper (if students are representing their learning on paper)

Organization: Individual

Scenarios:

The holiday season is right around the corner and you have some presents to purchase. You want to buy your family great gifts, but you also want to get a good price for the items. The following are some gifts you want to buy. The prices are taken from flyers obtained from different stores in your community.

For each of the purchase scenarios below, do the following:

- Determine the sale price of the items (rounded to the nearest cent).
- Represent the percents on a hundreds grid.
- Show the percents as fractions and decimals.
- Determine the total cost of the item once PST and GST are added.

Purchases: Round to the nearest tenth of a percent.

> 25% off a GPS system with preloaded North American maps
>
> Regular price: $199.99

45% off a new release of a movie

Regular price: $23.99

$5\frac{1}{2}$% off a 26″ LCD TV

Regular price: $419.99

Assessment:

The following rubric can be used to assess achievement of the mathematics learning outcomes.

Criteria	Meeting Expectations	Developing to Meet Expectations	Beginning to Meet Expectations	Incomplete
The student				
■ represents percents using grid paper	❒ accurately represents all three percents using grid paper	❒ accurately represents one or two percents using grid paper	❒ represents whole percents on grid paper but has errors with fractional percents	❒ does not represent percents using grid paper
■ converts between fractions, decimals, and percents	❒ consistently converts between fractions, decimals, and percents	❒ converts between fractions, decimals, and percents with few errors	❒ converts between fractions, decimals, and percents with many errors	❒ does not convert between fractions, decimals, and percents
■ solves problems that involve percents	❒ provides a clear explanation, using symbols and words, showing how he or she determined the sale price	❒ provides a general explanation, using symbols and words, showing how he or she determined the sale price	❒ provides a vague or minimal explanation showing how he or she determined the sale price	❒ provides no explanation showing how he or she determined the sale price

continued

Criteria	Meeting Expectations	Developing to Meet Expectations	Beginning to Meet Expectations	Incomplete
The student				
■ solves problems that involve combining percents	❒ clearly demonstrates how he or she determined the total cost when including PST and GST	❒ provides some explanation as to how he or she determined the total cost when including PST and GST	❒ provides minimal explanation as to how he or she determined the total cost when including PST and GST	❒ provides no explanation as to how he or she determined the total cost when including PST and GST
■ determines the discount	❒ accurately calculates the discount	❒ makes rounding errors that affect the final calculation of the discount	❒ makes calculation and rounding errors that affect the final calculation of the discount	❒ does not determine the discount
■ determines the sale price	❒ accurately calculates the sale price	❒ makes rounding errors that affect the calculation of the sale price	❒ makes calculation and rounding errors that affect the final calculation of the sale price	❒ does not determine the sale price
■ determines the new price with PST and GST added	❒ accurately determines the new price with PST and GST added	❒ makes rounding errors that affect the calculation of the new price with PST and GST added	❒ makes calculation and rounding errors that affect the final calculations of the new price with PST and GST added	❒ does not determine the new price with PST and GST added

NOTES

Number—8.N.4, 8.N.5

Enduring Understandings:

Ratios and rates are comparisons of two or more quantities.

Ratios can represent part-to-part quantities or part-to-whole quantities.

Ratios and rates can be used to solve proportional reasoning.

Percents, fractions, decimals, and ratios are all different representations of the same quantity.

General Learning Outcome:

Develop number sense.

Specific Learning Outcome(s):	**Achievement Indicators:**
8.N.4 Demonstrate an understanding of ratio and rate. [C, CN, V]	➛ Express a two-term ratio from a context in the forms 3:5 or 3 to 5. ➛ Express a three-term ratio from a context in the forms 4:7:3 or 4 to 7 to 3. ➛ Express a part-to-part ratio as a part-to-whole ratio (e.g., Given the ratio of frozen juice to water is 1 can to 4 cans, this ratio can be written as $\frac{1}{4}$, or 1:4, or 1 to 4 [part-to-part ratio]. Related part-to-whole ratios are $\frac{1}{5}$, or 1:5, or 1 to 5, which is the ratio of juice to solution, or $\frac{4}{5}$, or 4:5, or 4 to 5, which is the ratio of water to solution.). ➛ Identify and describe ratios and rates from real-life examples, and record them symbolically. ➛ Express a rate using words or symbols (e.g., 20 L per 100 km or 20 L/100 km). ➛ Express a ratio as a percent, and explain why a rate cannot be represented as a percent.
8.N.5 Solve problems that involve rates, ratios, and proportional reasoning. [C, CN, PS, R]	➛ Explain the meaning of $\frac{a}{b}$ within a context. ➛ Provide a context in which $\frac{a}{b}$ represents a ▪ fraction ▪ rate ▪ ratio ▪ quotient ▪ probability ➛ Solve a problem involving rate, ratio, or percent.

Prior Knowledge

Students may have had experience with the following:

- Demonstrating an understanding of fractions by using concrete and pictorial representations to
  - create sets of equivalent fractions
  - compare fractions with like and unlike denominators
- Demonstrating an understanding of ratio, concretely, pictorially, and symbolically
- Demonstrating an understanding of percent (limited to whole numbers), concretely, pictorially, and symbolically
- Solving problems involving percents from 1% to 100%

Related Knowledge

Students should be introduced to the following:

- Demonstrating an understanding of percents greater than or equal to 0%

Background Information

Rate, ratio, and proportion are three closely linked concepts:

- A *ratio* is a comparison of two or more quantities (e.g., 1 out of 4 people interviewed prefer . . .).
- A *rate* is a ratio that is often expressed as one quantity per unit of another quantity (e.g., 100 kilometres per hour).
- A *proportion* is a statement of equality between two ratios (e.g., $\frac{2}{3} = \frac{10}{15}$).

Each of these concepts is explained in detail below.

Ratios

A *ratio* is a comparison of two or more quantities.

Examples:

- To make this recipe, you need 2 kg of white flour for every 3 kg of whole wheat flour.
- At the airport, there is 1 taxi for every 20 people who arrive on a plane.

Some ratios, such as the ratio in the first example, are comparisons of one part of a whole (the amount of white flour) to another part of a whole (the amount of whole wheat flour). This is sometimes called a *part-to-part ratio*. For example, you buy 12 doughnuts—chocolate and glazed. The ratio of chocolate to glazed doughnuts is 5:7. But you could also compare the number of chocolate doughnuts to the total number of doughnuts (5:12). This is sometimes called a *part-to-whole ratio*.

In the ratio 5:7, the numbers 5 and 7 are called the *terms* of the ratio. The first term is 5 and the second term is 7. Ratios should be taught in the context of everyday situations and students should have opportunities to use ratios with concrete materials (e.g., making orange juice from concentrate requires 3 cans of water: 1 can concentrate).

It is common to name ratios using fractions. Doing so, however, may cause confusion for Middle Years students. To avoid any misconceptions, two important points need to be understood about ratio:

- Ratio is one of the meanings of fraction. When you say, $\frac{3}{4}$ of the flowers in your garden are annuals, you are comparing the annuals to all the flowers (a part-to-whole ratio), and this is as valid as saying 3 out of 4 flowers are annuals. However, some ratios can never be written as fractions. For example, the ratio 9:0 would result in the denominator of 0, which is mathematically incorrect
- If you have 1 taxi for 5 people, the ratio 1:5 is not a fraction. The ratio can be written as $\frac{1}{5}$, but the 5 people are not the whole and the taxi is not 1 part of that whole.
- Although part-to-part ratios are sometimes written as fractions, it is important not to change these fractions to percents. For example, you may see the ratio of boys to girls written as 4:5 or $\frac{4}{5}$, but it would be mathematically incorrect to say 80% are boys. Reinforce that fractions represent part-to-whole relationships.

To avoid confusion about the concept of naming ratios using fractions, use the traditional notation (e.g., 2:3) or the words (2 to 3) when expressing ratios until students have a clear understanding of ratio. Then extend the concept to include the link between ratios and fractions.

Rates

Sometimes a ratio can also be a rate. A *rate* is a comparison that relates the measures for two different types of quantities. For each measure, the unit is different and is included when writing the ratio or rate.

Examples:

- All prices are rates and ratios (e.g., 49 cents each, 3 for a dollar, $1.99 per kilogram).
- The comparison of time to distance is a rate (e.g., driving at 65 kilometres per hour).
- Changes between two units of measure are also rates or ratios (e.g., centimetres per metre, millilitres per litre, map scales).
- Unit pricing is a common rate used to compare value when purchasing items (e.g., 35¢ per can, $1.24 per kilogram). It is, however, commonly misunderstood by both students and consumers.

Proportion

A *proportion* is a statement of the equality between two ratios.

Examples:

- A rectangle is drawn on grid paper and you want to copy it, but triple its size.
- The florist has a special on bouquets. Customers receive two free carnations for every rose purchased. You want to buy one of these bouquets for your mother. If you want 6 roses in the bouquet, how many carnations will it include?

Each of these situations deals with proportion. To solve the question of the flowers, students would set up a proportion statement such as the following:

Roses:Carnations		Roses:Carnations
6:?	=	1:2

Students should be very secure in using this type of notation before they are introduced to the fraction symbol for proportion.

Although proportions and equivalent fractions appear to be the same thing, they are not. Equivalent fractions are different symbols for the same amount. If you colour $\frac{1}{2}$ a piece of paper and fold it so that it now shows $\frac{2}{4}$, you still have the same amount coloured ($\frac{1}{2} = \frac{2}{4}$). On the other hand, if you buy two bouquets of flowers and one has 1 rose and 2 carnations and the other has 2 roses and 4 carnations, the total number of flowers is different, but the ratio of roses to carnations is the same (1:2 = 2:4) and, therefore, proportional.

Mathematical Language

equivalent fraction

part-to-part ratio

part-to-whole ratio

proportion

rate

three-term ratio

two-term ratio

unit price

unit rate

Assessing Prior Knowledge

Materials: Pattern blocks, BLM 8.N.4.1: Ratio Pre-Assessment

Organization: Individual

Procedure:

1. Tell students that in the next few lessons they will be learning about rates, ratios, and proportions; however, you first need to find out what they already know about ratios.
2. Ask students to complete BLM 8.N.4.1: Ratio Pre-Assessment.

Observation Checklist

☑ Observe students' responses to determine whether they can do the following:
- ❒ Represent ratios concretely.
- ❒ Represent ratios pictorially.
- ❒ Represent ratios symbolically.
- ❒ Solve problems involving ratios.

Suggestions for Instruction

- **Express a two-term ratio from a context in the forms 3:5 or 3 to 5.**
- **Express a three-term ratio from a context in the forms 4:7:3 or 4 to 7 to 3.**

Materials: Red, white, and blue poker chips (or any other item that can be used to create a ratio), 6 glass beakers or jars, math journals

Organization: Small group/whole class

Procedure:

1. Tell students that in the next few lessons they will be learning about ratios.
2. Place a variety of the three colours of poker chips into beakers. Ensure that each beaker has 10 chips in total.

3. Divide students into small groups. Ask each group to answer the following questions and be prepared to share responses with the rest of the class:
   - How could you compare the number of blue chips to the number of red chips?
   - How could you compare the number of blue chips to the number of white chips?
   - How could you compare the number of red chips to the number of white chips?
   - How could you compare the number of blue chips to the total number of chips?
   - How could you compare the number of red chips to the total number of chips?
   - How could you compare the number of white chips to the total number of chips?
   - How could you compare the number of blue and red chips to the number of white chips?
   - How could you compare the number of red and white chips to the number of blue chips?
   - How could you compare the number of blue and red chips to the total number of chips?
   - How could you compare the number of blue chips to the number of red chips to the number of white chips?
4. After groups have shared their results with the class, define *ratio*. Put the definition on the bulletin board. Rewrite the ratio for the first statement in step 3 above. Tell students that this ratio is a two-term ratio. Rewrite the ratio for the last statement in step 3 above. Tell students that this ratio is a three-term ratio.
5. Have students return to their groups. Ask them to decide on a definition for *two-term ratios* and *three-term ratios* and share their definitions with the class.
6. Have students explain the following in their math journals, using words and diagrams: a ratio, a two-term ratio, and a three-term ratio.

Observation Checklist

☑ Use students' math journal responses to determine whether they can do the following:

- ❐ Explain two-term ratio using words and diagrams.
- ❐ Explain three-term ratio using words and diagrams.

Suggestions for Instruction

- **Express a part-to-part ratio as a part-to-whole ratio (e.g., Given the ratio of frozen juice to water is 1 can to 4 cans, this ratio can be written as $\frac{1}{4}$, or 1:4, or 1 to 4 [part-to-part ratio]. Related part-to-whole ratios are $\frac{1}{5}$, or 1:5, or 1 to 5, which is the ratio of juice to solution, or $\frac{4}{5}$, or 4:5, or 4 to 5, which is the ratio of water to solution.).**
- **Identify and describe ratios and rates from real-life examples and record them symbolically.**
- **Express a ratio as a percent and explain why a rate cannot be represented as a percent.**

Materials: 6 cans of a variety of frozen fruit juice concentrate, 6 pitchers, math journals

Organization: Small group/whole class

Procedure:

1. Divide students into small groups, and ask them to follow the directions to make fruit juice from frozen concentrate.
   - Write the ratio of cans of frozen fruit juice concentrate to the number of cans of water. (Usually 1:3 or 1:4)
   - Write the ratio of cans of frozen fruit juice concentrate to the total number of cans of juice made. (Either 1:4 or 1:5)
2. Ask students to answer the following questions:
   - How does the ratio of cans of frozen fruit juice concentrate to the number of cans of water reflect a part-to-part ratio?
   - How does the ratio of frozen fruit juice concentrate to the total number of cans of juice made reflect a part-to-whole ratio?
3. Explain that a part-to-whole ratio can also be written as a percent.
   - Ask students to express the part-to-whole ratio of cans of frozen juice concentrate to the total solution of juice made as a percent. (1:5 or 20%)
   - Ask students to express the ratio of water to the total solution. (4:5 or 80%)

Note: Although part-to-part ratios can be written as a fraction for proportional equivalency (e.g., one can of concentrate to four cans of water [1:4] can be increased 10-fold to 10:40 or $\frac{1}{4} = \frac{10}{40}$), reinforce with students that fractions represent part-to-whole relationships. Conversion of ratios to a percent makes sense only in the part-to-whole context.

4. Ask students to share their responses with the class. As a class, discuss that part-to-part ratios compare different parts of a group to each other and part-to-whole ratios compare one part of a group to the whole group. So, in this case, the two parts of the groups are the frozen concentrate and the water. The total is the 4 or 5 cans of total liquid the mixture would make.
5. Ask students to write the ratio of boys to girls in the class and the ratio of boys to the total student population.
6. Ask students: How does the ratio of boys to girls reflect a part-to-part ratio and the ratio of boys to the total student population reflect a part-to-whole ratio? Discuss responses as a class.
7. Have students discuss the following in their math journals:
   - Describe, using words and diagrams, part-to-part ratios and part-to-whole ratios.
   - Use real-life examples to enhance the description of ratios.

Observation Checklist

☑ Use students' math journal responses to determine whether they can do the following:
- ❒ Describe part-to-part ratios.
- ❒ Describe part-to-whole ratios.
- ❒ List real-life examples of ratios.

Suggestions for Instruction

- **Identify and describe ratios and rates from real-life examples, and record them symbolically.**
- **Express a rate using words or symbols (e.g., 20 L per 100 km or 20 L/100 km).**
- **Express a ratio as a percent, and explain why a rate cannot be represented as a percent.**

Materials: Poster paper, flyers, the Internet, math journals

Organization: Whole class/individual

Procedure:

1. Write the following examples of rates on the whiteboard:
   100 km/h, 70 beats/min, $1.69/100 g, $9.50/h
2. Ask students whether they can tell you what the rates are in the examples provided. (They may say speed, heart rate, money, wages, and so on.)
3. Explain to students that all of them are examples of rates. Rates are special ratios. *Rates* compare two quantities measured in different units. Ask students what the different units are in the examples provided (i.e., distance and time, speed of heart rate and time, money and mass, and money and time). For this reason, rates can never be expressed as a percent.
4. Ask students where they may have seen the examples of rates in the real world.
5. Ask students whether they have seen any other rates. (Generate a list from student responses.)
6. Write the following examples or rates on the whiteboard:
   400 km per 4 h, 140 beats per 2 min, $16.90 per 1000 g, $28.50 per 3 h
7. Ask students whether they see any difference between the original list of rates and the new list of rates. (You want to end up with the understanding that both lists are rates, but the first list is unit rates. A *unit rate* is a rate in which the second term is one.)
8. Have students find examples of rates (e.g., in shops, in flyers, on the Internet) or develop their own posters displaying rates. Have students describe, in their math journals, the meaning of their sample rates using words and symbols.

Observation Checklist

☑ Use students' math journal responses to determine whether they can do the following:

- ❒ Expresses rates using words or symbols.
- ❒ Make connections between rate and ratio.
- ❒ Make connections between rate and real-life examples.

Suggestions for Instruction

- **Explain the meaning of $\frac{a}{b}$ within a context.**
- **Provide a context in which $\frac{a}{b}$ represents a**
  - **fraction**
  - **rate**
  - **ratio**
  - **quotient**
  - **probability**

Materials: BLM 8.N.4.2: Meaning of $\frac{a}{b}$?, chart paper, math journals

Organization: Whole class/small group

Procedures:

1. Explain to students that $\frac{a}{b}$ can represent a fraction, rate, ratio, quotient, or probability depending on the context.
2. Facilitate a short class discussion to ensure that students understand the meaning of each of these terms: *fraction, rate, ratio, quotient,* and *probability*.
3. Inform students that they will be playing an adapted version of the game *Four Corners*.
   - Explain that five spaces in the room are labelled *fraction, rate, ratio, quotient*, and *probability*.
   - Present students with one scenario from BLM 8.N.4.2: Meaning of $\frac{a}{b}$?
   - Allow students some thinking time, and then have them write on a piece of paper which of the five meanings of $\frac{a}{b}$ the scenario represents.
   - Ask students to go to the appropriate space in the room.
   - Have all students in a given space work together to come up with an explanation of why they made their selection in order to try to convince the other groups to change their minds.
   - All groups have a chance to share their reasons with the rest of the class. Students may move from group to group as many times as they like if they have been convinced by another group.
   - Afterward, reveal which is the correct meaning of the given context, and facilitate further discussion with the class.
4. Repeat the process with the remaining scenarios from BLM 8.N.4.2: Meaning of $\frac{a}{b}$?
5. In their math journals, students use the ratio 2 to 3 in a way that means fraction, rate, ratio, quotient, and probability.

Observation Checklist

☑ Observe students and use their math journals to determine whether students can do the following:

- ❒ Explain when $\frac{a}{b}$ represents a fraction.
- ❒ Explain when $\frac{a}{b}$ represents a rate.
- ❒ Explain when $\frac{a}{b}$ represents a ratio.
- ❒ Explain when $\frac{a}{b}$ represents a quotient.
- ❒ Explain when $\frac{a}{b}$ represents a probability.
- ❒ Provide a scenario in which $\frac{a}{b}$ represents a fraction.
- ❒ Provide a scenario in which $\frac{a}{b}$ represents a rate.
- ❒ Provide a scenario in which $\frac{a}{b}$ represents a ratio.
- ❒ Provide a scenario in which $\frac{a}{b}$ represents a quotient.
- ❒ Provide a scenario in which $\frac{a}{b}$ represents a probability.
- ❒ Communicate mathematically

Suggestions for Instruction

- **Solve a problem involving rate, ratio, or percent.**

Materials: BLM 8.N.4.3: Problem Solving, chart paper, math journals

Organization: Individual/small group

Procedure:

1. Explain to students that they will be solving problems using their understanding of rate, ratio, fractions, and percent.
2. Have students form small groups, and provide each group with a copy of BLM 8.N.4.3: Problem Solving. Ask students to solve the problems presented, record the problem-solving process on chart paper, and be prepared to explain their results.
3. As groups present their results, identify the different strategies that were used to solve the problems.

4. Ask students to respond to the following in their math journals:

   There are two hundred students at Santa B Middle School in North Pole City. If 42% of the students are female, what is the ratio of female students to male students? Present your answer in a variety of forms. (42:58, 84:116, 21/29)

Observation Checklist

☑ Use students' math journal responses to determine whether students can do the following:

- ❒ Solve problems involving rate, ratio, or percent.
- ❒ Apply problem-solving strategies.
- ❒ Communicate solutions to problems using mathematical language.
- ❒ Use prior knowledge to reason mathematically.

Suggestions for Instruction

- **Express a two-term ratio from a context in the forms 3:5 or 3 to 5.**
- **Express a three-term ratio from a context in the forms 4:7:3 or 4 to 7 to 3.**
- **Express a part-to-part ratio as a part-to-whole ratio (e.g., Given the ratio of frozen juice to water is 1 can to 4 cans, this ratio can be written as $\frac{1}{4}$, or 1:4, or 1 to 4 [part-to-part ratio]. Related part-to-whole ratios are $\frac{1}{5}$, or 1:5, or 1 to 5, which is the ratio of juice to solution, or $\frac{4}{5}$, or 4:5, or 4 to 5, which is the ratio of water to solution.).**
- **Solve a problem involving rate, ratio, or percent.**

Materials: BLM 5–8.9: Centimetre Grid Paper, pencil crayons of various colours, chart paper, poster paper

Organization: Pairs

Procedure:

1. Tell students that they will be creating designs on a 5 × 5 grid using three colours of pencil crayons.
2. Have students select a partner to work with.
3. Ask each pair to decide on three colours that both partners will use to make their designs.

4. Ask students to decide on a ratio of the three colours they will use in their designs. Both students in a pair should use the same ratio.
5. Once the designs are complete, have students predict the number of squares that would be in each colour in their designs if they were to use a 10 × 10 grid and follow the same design (essentially, scaling up their diagrams). Have students exchange designs with their partners and create each others' designs using a 10 × 10 grid.
6. Ask students to create a poster that includes
   - their designs
   - the colours expressed as fractions, as ratios, as percents, and in words

Observation Checklist

☑ Observe students' responses to determine whether they can do the following:

- ❒ Solve problems involving rate, ratio, or percent.
- ❒ Apply problem-solving strategies.
- ❒ Correctly express a two-term ratio.
- ❒ Correctly express a three-term ratio.
- ❒ Correctly express a ratio as part-to-part and part-to-whole.
- ❒ Correctly express a ratio in fractional, ratio, and percent notation, or in words.

Putting the Pieces Together

Grade 8 Farewell

Introduction:

Schools that go up to Grade 8 often have a celebration at the end of the year before students head to high school. Some schools call it a Grade 8 graduation; others call it a Grade 8 farewell. This learning task allows students to participate in a real-life situation in which they volunteer to plan the Grade 8 farewell for a school.

Purpose:

Students will use skills in collecting data and solving proportions.

Curricular Links: LwICT, ELA

Materials/Resources: Access to *Microsoft* PowerPoint, the Internet

Organization: Individual or small group

Scenario:

- You are organizing the school's Grade 8 farewell. There will be 75 people attending.
- You will be planning everything related to the food.
  - You know that you will be having pizza for the main course. What types of pizza will you need to order, and how much?
  - How much dessert will you need?
  - How many drinks will you need?
  - How much cutlery will you need? how many glasses and plates? how many napkins?
  - Can you think of anything else you might need?
- You will be planning the balloon decorations.
  - How many balloons will you be using?
  - How many balloons are in a package?
- You will be arranging the DJ who will play the music for the farewell.
- You will need to determine the final cost for the entire farewell and then determine the price per student so that you know how much to charge each student for attending the farewell.

Procedure:

1. Create a survey to determine what types of pizzas to order. Then determine how many of each type need to be ordered based on the number of slices in the large pizza size.
2. Survey one class and base your decisions on that population.
3. If the desserts come in amounts of one dozen, how many will you need to order? (Is everyone getting one dessert or more than one?)
4. If the drinks come in cases of 24, how many will you need to order? (Is everyone getting one drink or more than one?)
5. If the cutlery, paper plates, glasses, and napkins come in packages of 24, how many of each will you need to purchase?
6. Research DJs in your area. Select one DJ and include the price. You may have to call for a price.
7. Once you have collected all the information you need, put it into a PowerPoint so that you can present your work to your peers.
8. All the math must be done showing ratios/proportions.
9. If you ordered more or less, you need to explain your reasoning behind it.
10. Include the final cost per student and the determined ticket price for students to attend the farewell.

11. Include an assumptions page, since you will have to make many assumptions as you work through this planning process.

Assessment:

The following rubric can be used to assess achievement of the mathematics learning outcomes.
Note: Other rubrics may be added to assess LwICT and ELA learning outcomes.

Criteria	Meeting Expectations	Developing to Meet Expectations	Beginning to Meet Expectations	Incomplete
The student				
■ demonstrates an understanding of surveys	❐ clearly states survey questions ❐ provides a clearly organized survey tracking sheet ❐ keeps a tally that is easy to read ❐ provides a conclusion that clearly explains why the types of pizzas and the number of each type were ordered	❐ states survey questions ❐ provides a somewhat organized survey tracking sheet ❐ keeps a tally ❐ provides a conclusion that partially explains why the types of pizzas and the number of each type were ordered	❐ vaguely states survey questions ❐ provides a disorganized survey tracking sheet ❐ keeps a vague tally ❐ provides a conclusion but no explanation, just states the types of pizzas and the number of each type ordered	❐ does not conduct the survey
■ solves problems that involve rates, ratios, and proportional reasoning	❐ provides an explanation of all the rates, ratios, and proportional reasoning used	❐ provides an explanation of some of the rates, ratios, and proportional reasoning used	❐ provides an explanation of few of the rates, ratios, and proportional reasoning used	❐ provides no explanation of the rates, ratios, and proportional reasoning used
■ makes assumptions in order to solve problems	❐ clearly explains rationalizations behind the assumptions made	❐ partially explains rationalizations behind the assumptions made	❐ vaguely explains rationalizations behind the assumptions made	❐ does not explain rationalizations behind the assumptions made

Extension:

Students can use a pie chart to show the allocation of expenses and use this pie chart to predict the expenses of a similar party with *x* number of people attending.

NOTES

Number—8.N.6, 8.N.8

Enduring Understandings:

Fractions represent parts of a whole or part of a group.

Percents, fractions, ratios, and decimals are different representations of the same quantity.

Fractions can represent division.

Multiplication does not always make a bigger group.

The principles of operations used with whole numbers also apply to operations with decimals, fractions, and integers.

General Learning Outcome:

Develop number sense.

Specific Learning Outcome(s):	**Achievement Indicators:**
8.N.6 Demonstrate an understanding of multiplying and dividing positive fractions and mixed numbers, concretely, pictorially, and symbolically. [C, CN, ME, PS]	➛ Identify the operation(s) required to solve a problem involving positive fractions. ➛ Provide a context involving the multiplying of two positive fractions. ➛ Provide a context involving the dividing of two positive fractions. ➛ Express a positive mixed number as an improper fraction and a positive improper fraction as a mixed number. ➛ Model multiplication of a positive fraction by a whole number, concretely or pictorially, and record the process. ➛ Model multiplication of a positive fraction by a positive fraction, concretely or pictorially, and record the process. ➛ Model division of a positive fraction by a whole number, concretely or pictorially, and record the process. ➛ Generalize and apply rules for multiplying and dividing positive fractions, including mixed numbers. ➛ Solve a problem involving positive fractions taking into consideration order of operations (limited to problems with positive solutions).

continued

Specific Learning Outcome(s):	**Achievement Indicators:**
8.N.8 Solve problems involving positive rational numbers. [C, CN, ME, PS, R, T, V]	➛ Identify the operations(s) required to solve a problem involving positive rational numbers. ➛ Determine the reasonableness of an answer to a problem involving positive rational numbers. ➛ Estimate the solution and solve a problem involving positive rational numbers. ➛ Identify and correct errors in the solution to a problem involving positive rational numbers.

Prior Knowledge

Students may have had experience with the following:

- Demonstrating an understanding of multiplication (2-digit numerals by 2-digit numerals) to solve problems
- Relating decimals to fractions (tenths, hundredths, thousandths)
- Demonstrating an understanding of factors and multiples by
  - determining multiples and factors of numbers less than 100
  - identifying prime and composite numbers
  - solving problems involving factors or multiples
- Relating improper fractions to mixed numbers
- Demonstrating an understanding of multiplication and division of decimals involving
  - 1-digit whole-number multipliers
  - 1-digit natural number divisors
  - multipliers and divisors that are multiples of 10
- Explaining and applying the order of operations, excluding exponents (limited to whole numbers)
- Demonstrating an understanding of the addition, subtraction, multiplication, and division of decimals to solve problems (for more than 1-digit divisors or 2-digit multipliers, the use of technology is expected)
- Demonstrating an understanding of the relationship between repeating decimals and fractions, and terminating decimals and fractions
- Demonstrating an understanding of adding and subtracting positive fractions and mixed numbers, with like and unlike denominators, concretely, pictorially, and symbolically (limited to positive sums and differences)

Background Information

Multiplying and Dividing Fractions

Having a concrete, pictorial, and symbolic understanding of what happens when multiplying and dividing fractions enables students to have a better conceptual understanding of how and why the various methods work. Students should apply their prior knowledge of fractions, and of performing operations on whole and decimal numbers, when learning about multiplying and dividing fractions.

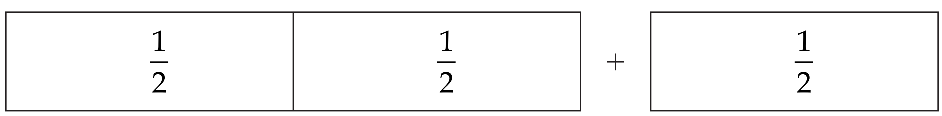
It is essential to give students time to develop their conceptual understanding of multiplying and dividing fractions. Once students understand what is happening using concrete and pictorial representations, they can develop and apply a symbolic method for multiplying and dividing fractions.

Meanings of Multiplication

It is important that students are able to work flexibly with the various meanings of multiplication.

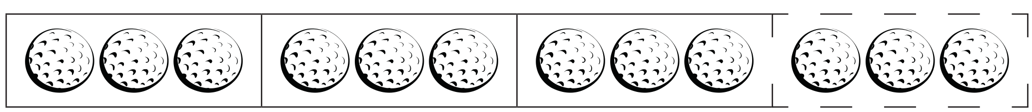
- **Multiplication as repeated addition:**

  $3 \bullet \frac{1}{2} = \frac{1}{2} + \frac{1}{2} + \frac{1}{2} = 1\frac{1}{2}$



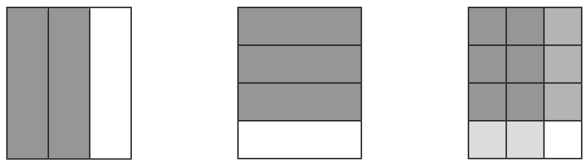
- **Multiplication as equal sets or groups:**

  $\frac{3}{4} \bullet 12$ (think of $\frac{3}{4}$ of a set of 12 objects) = 9



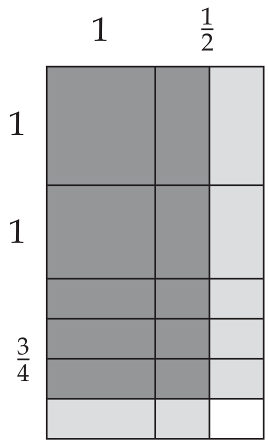
- **Multiplication as a rectangular area:**

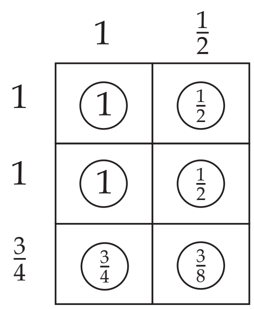
$$\frac{2}{3} \quad \bullet \quad \frac{3}{4} \quad = \quad \frac{6}{12} \text{ or } \frac{1}{2}$$



Note: At first, students can model this using concrete materials or on square grid paper. As students learn to trust the count and become more skilled, they may be able to draw the multiplications free-hand.

$1\frac{1}{2} \bullet 2\frac{3}{4}$





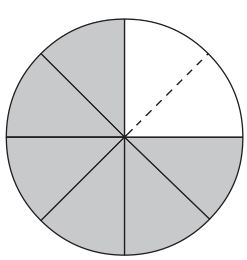
$$\begin{aligned} 1\frac{1}{2} \bullet 2\frac{3}{4} &= 1 + 1 + \frac{1}{2} + \frac{1}{2} + \frac{3}{4} + \frac{3}{8} \\ &= 3 + \frac{3}{4} + \frac{3}{8} \\ &= 3 + \frac{6}{8} + \frac{3}{8} \\ &= 3 + \frac{9}{8} \\ &= 3 + 1\frac{1}{8} \\ &= 4\frac{1}{8} \end{aligned}$$

Meanings of Division

It is important that students are able to work flexibly with the various meanings of division.

- **Division as equal sharing:**

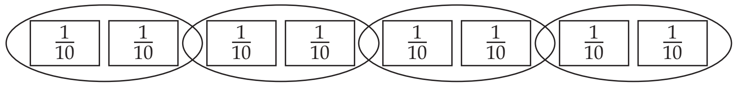
  $\frac{3}{4}$ of a pizza shared among 6 people:



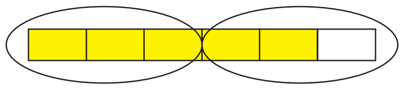
$$\frac{3}{4} \div 6 = \frac{1}{8}$$

- **Division as equal grouping:**

$$\frac{8}{10} \div 4 = \frac{2}{10} \text{ or } \frac{1}{5}$$

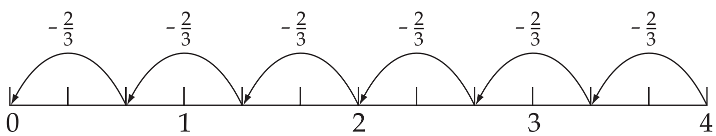


$$\frac{5}{6} \div \frac{1}{2} = 1\frac{2}{3}$$



- **Division as repeated subtraction:**

$$4 \div \frac{2}{3} = 6$$



Mathematical Language

denominator

fraction

improper fraction

mixed number

numerator

order of operations

proper fraction

rational numbers

reciprocal

Assessing Prior Knowledge

A pre-assessment will help determine students' level of understanding of fractions and provide a baseline for instruction. It will direct teaching based on students' learning needs. Make sure the pre-assessment is similar to the post-assessment, thereby enabling students to focus on areas for improvement and then providing them with opportunities to show that they have developed an understanding of the concepts.

Materials: 7 pieces of chart paper per group, math journals

Organization: Small group/whole class/individual

Procedure:

1. Tell students that they will be learning to multiply and divide fractions over the next few lessons; however, you first need to determine what they already know about fractions.
2. Have students form small groups. Provide each group with the following list and one sheet of chart paper for each concept in the list: factors, multiples, prime numbers, composite numbers, improper fractions and mixed numbers, adding fractions, and subtracting fractions.
3. Have each group explain, to the best of their ability, each concept, using words, diagrams, symbols, or concrete items. Students need to record their work on the chart paper and be prepared to share their group's thoughts to the class.
4. Review students' responses, providing opportunities for questions and discussions. Encourage groups to add to their papers as needed when listening to other groups.
5. In their math journals, have students
    - write the following at the top of the page: factors, multiples, prime numbers, composite numbers, improper fractions and mixed numbers, adding fractions, and subtracting fractions
    - summarize their understanding of the concepts based on the learning activity in which they participated

Observation Checklist

☑ Use students' math journal responses to determine whether students can do the following:

- ❒ List factors of numbers less than 100.
- ❒ Identify multiples of different numbers.
- ❒ Explain what prime numbers are.
- ❒ Explain what composite numbers are.
- ❒ Explain the relationship between improper fractions and mixed numbers.
- ❒ Explain how to add fractions.
- ❒ Explain how to subtract fractions.

Suggestions for Instruction

- **Express a positive mixed number as an improper fraction and a positive improper fraction as a mixed number.**

Materials: Pattern blocks or 4 or 5 copies of cut paper fraction bars per student (see BLM 5–8.12: Fraction Bars), BLM 8.N.6.1: Mixed Numbers and Improper Fractions

Organization: Whole class/individual

Procedure:

1. Begin by having students show various representations for one (e.g., $\frac{4}{4}$, $\frac{10}{10}$).
2. Provide students with 5 halves. Ask them:
   - What fraction do these pieces represent? ($\frac{5}{2}$)
   - What type of fraction is $\frac{5}{2}$? (Improper fraction)
   - Is there another way $\frac{5}{2}$s can be represented? (As a mixed number: $2\frac{1}{2}$)
   - Can you put the pieces together to show $\frac{5}{2}$ as $2\frac{1}{2}$?
3. Ask students to show you $\frac{8}{6}$. (They need to have 8 sixth pieces.) Walk around the classroom to ensure that students have the correct amount. Have students represent $\frac{8}{6}$ as a mixed number. ($1\frac{2}{6}$) Some may be able to simplify it to $1\frac{1}{3}$.
4. Ask students whether they can come up with a strategy for converting improper fractions to mixed numbers. Facilitate a class discussion about their strategies.
5. Write $1\frac{3}{4}$ on the whiteboard. Ask students to read the mixed number.

6. Have students model this number using their fraction materials.
7. Ask whether they can express this number as an improper fraction.
8. Repeat, using $2\frac{1}{3}$.
9. Ask students whether they can come up with a strategy for converting improper fractions to mixed numbers. Facilitate a class discussion about their strategies.
10. Ask students to convert the following mixed numbers to improper fractions, using a method of their choosing: $2\frac{2}{3}$, $3\frac{1}{2}$, $4\frac{4}{5}$. Review one at a time. Have individual students demonstrate the solution on the whiteboard. Allow opportunities for students to give feedback and ask questions.
11. Provide students with BLM 8.N.6.1: Mixed Numbers and Improper Fractions. Have them respond to the questions individually.

Observation Checklist

☑ Observe students' responses to determine whether they can do the following:

- ❒ Understand that a mixed number and an improper fraction are two equivalent representations.
- ❒ Convert a mixed number to an improper fraction.
- ❒ Convert an improper fraction to a mixed number.
- ❒ Use mental mathematics and estimation strategies.

Suggestions for Instruction

- **Express a positive mixed number as an improper fraction and a positive improper fraction as a mixed number.**

Materials: BLM 5–8.5: Number Cards with the zeros removed, BLM 8.N.6.2: Mixed Number War

Organization: Pairs/whole class

Procedure:

Note: Since this is a game of speed, have students paired up equally in their ability to convert between mixed numbers and improper fractions.

1. Tell students that they will be learning a new game called *Mixed Number War.* Explain how the game is played:
   - Player A flips one card and, without looking at the card, places it in either the numerator or the denominator place.

- Player B flips a card and, without looking at the card, places it in the remaining space.
- If a proper fraction is made, the first one to slap a hand onto the fraction wins the cards.
- If an improper fraction is made, the first one to name it correctly as a mixed number wins the cards.
- If a player slaps the cards when the fraction is not a proper fraction or incorrectly names the mixed number, the player's partner wins the cards.

2. Demonstrate to the class how the game is played, and have students play.

Observation Checklist

☑ Observe students' responses to determine whether they can do the following:

- ❒ Understand that a mixed number and an improper fraction are two equivalent representations.
- ❒ Recognize proper and improper fractions.
- ❒ Convert an improper fraction to a mixed number.

Suggestions for Instruction

- **Determine the reasonableness of an answer to a problem involving positive rational numbers.**
- **Estimate the solution and solve a problem involving positive rational numbers.**

Materials: White paper, highlighters, pens of different colours, math journals

Organization: Individual/whole class

Procedure:

1. Begin the lesson by telling students that they will be reviewing the addition and subtraction of proper fractions, improper fractions, and mixed numbers.
2. Ask students what they remember about adding and subtracting proper fractions, improper fractions, and mixed numbers. Provide students with sheets of white paper and have them brainstorm.

Note: *Rational numbers* are any numbers that can be written in fraction form. The denominator cannot be zero. Rational numbers can be added, subtracted, multiplied, and divided. This is a perfect time to review adding and subtracting fractions from Grade 7.

3. Once students have done this individually, ask them to take out a highlighter and pens of different colours. Ask students to share their ideas with the class, and make a large class web of their suggestions. Have students highlight the information they had put down on their sheets that is correct and add new information they didn't have on their sheets.
4. Collect students' work to check for any misconceptions in students' thinking, and provide clarification as needed.
5. In their math journals, have students demonstrate their understanding of adding and subtracting proper fractions, improper fractions, and mixed numbers, using words, symbols, and/or diagrams.

Observation Checklist

☑ Use students' math journal responses to determine whether they can do the following:

- ❒ Add and subtract proper fractions.
- ❒ Add and subtract improper fractions and mixed numbers.
- ❒ Create equivalent fractions where needed.
- ❒ Communicate mathematically.

Suggestions for Instruction

- **Identify the operation(s) required to solve a problem involving positive rational numbers.**
- **Determine the reasonableness of an answer to a problem involving positive rational numbers.**
- **Estimate the solution and solve a problem involving positive rational numbers.**

Materials: BLM 8.N.6.3: Decimal Addition Wild Card, BLM 5–8.5: Number Cards

Organization: Pairs

Procedure:

1. Before students begin this learning activity, prepare enough sets of number cards for the class (sets contain four of each digit) by copying them on paper or card stock. Randomly mark (with a marker or sticker) the corner of the number side of four cards from each set—these will be the wild cards.

2. Explain to students that they will be learning a new game called *Decimal Addition Wild Card*. Explain the following rules, and demonstrate a round for the class:
   - The object of the game is to have the greatest sum after each round of play.
   - Shuffle the deck (including the wild cards) and place the cards face-down in a pile or spread out.
   - Player A selects one card, and then decides where to place the card on the top of his or her recording sheet.
   - Player B does the same.
   - Play continues in this manner until all spaces are filled on the top of each player's recording sheet.
   - Both players fill out their chart at the bottom of the recording sheet accordingly, and circle the player with the highest sum after the round.
   - The player with the greatest number of highest sums after nine rounds of play wins.

   Similar games could be played with any of the operations.

Note: If players select a wild card, they have two options:

- They may play the number shown on the card as they would any regular card.
- They can choose to swap the position of two cards, and select a new card.

Observation Checklist

☑ Observe students' responses to determine whether they can do the following:

- ❒ Use mental mathematics and estimation skills.
- ❒ Reason mathematically in order to place numbers.
- ❒ Add (subtract, multiply, divide) positive decimal numbers.

Suggestions for Instruction

- **Model multiplication of a positive fraction by a whole number, concretely or pictorially, and record the process.**
- **Generalize and apply rules for multiplying and dividing positive fractions, including mixed numbers.**

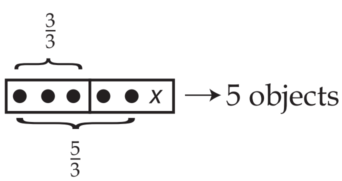
Materials: Pattern blocks, fraction bars, grid paper, square tiles, empty number lines, math journals

Organization: Whole class/individual

Procedure:

> **Note:** Whenever students are solving a problem, they are expected to explain what they did to get the answer, why they solved the problem that way, and why they think the solution makes sense.

1. Tell students that in the next few lessons they will be learning how to multiply and divide fractions concretely, pictorially, and symbolically.
2. Have one student neatly record onto a transparency all multiplication sentences and answers talked about in class.
3. Review multiplication by asking students the following question: What is *multiplication*? Encourage a variety of responses.
4. Provide students with a variety of manipulatives. If necessary, review with students the fractional representation of the fraction bars and pattern blocks.
5. Ask students to use the manipulatives to show $5 \bullet \frac{1}{6}$ and $3 \bullet \frac{5}{3}$. Some students may show manipulatives as groups (e.g., five groups of $\frac{1}{6}$), as repeated addition (e.g., $\frac{1}{6} + \frac{1}{6} + \frac{1}{6} + \frac{1}{6} + \frac{1}{6}$), or as equal sets (e.g., $\frac{5}{3}$ of a set of 3 objects).

   Discuss each of the three representations of multiplication.
6. Have students make up their own multiplication sentences that use a fraction and a whole number. Have them model this multiplication using all three meanings for multiplication.
7. Discuss the various models with the class, encouraging a variety of responses:
   - Which model is best?
   - Which model is most clear?
   - Which model is easiest to understand?
8. Ask the recorder to share the multiplication sentences and answers discussed in class.
9. Have the class discuss any patterns they see. Ask students whether they can come up with a rule for multiplying fractions and whole numbers based on what they have been doing. (You may have to demonstrate that a whole number is the same as any number over 1 in fraction form.)
10. Have students answer the following question in their math journals:
    - When multiplying a whole number by a proper fraction, what can you say about the size of the product in comparing it to the two factors? (The product would be less than the whole number factor and greater than the fraction factor.)
    - Use diagrams, words, and symbols to support your response.

Observation Checklist

☑ Observe students' responses to determine whether they can do the following:

- ❒ Connect the concept of multiplying whole numbers to that of fractions.
- ❒ Represent multiplication in a variety of ways.
- ❒ Multiply proper fractions by whole numbers concretely, pictorially, and symbolically.
- ❒ Communicate mathematically.
- ❒ Recognize a pattern in order to generalize a rule.

☑ Use students' math journal responses to determine whether they can do the following:

- ❒ Recognize that when multiplying a whole number by a fraction, the product would be less than the whole number and greater than the fraction factor.
- ❒ Demonstrate the multiplication of a proper fraction by a whole number in a variety of ways.
- ❒ Communicate mathematically.

Suggestions for Instruction

- **Model division of a positive fraction by a whole number, concretely or pictorially, and record the process.**
- **Generalize and apply rules for multiplying and dividing positive fractions, including mixed numbers.**

Materials: Pattern blocks, fraction bars, grid paper, square tiles, empty number lines, math journals

Organization: Whole class/individual

Procedure:

1. During the class discussion of division sentences, have one student neatly record onto a transparency all responses.
2. Review division by asking students the following question: What is *division*? Encourage a variety of responses.

3. Provide students with a variety of manipulatives. If necessary, review with students the fractional representation of the fraction bars and pattern blocks.
4. Ask students to use the manipulatives to show $\frac{5}{6} \div 5$ and $\frac{1}{5} \div 2$. (Some students may show equal sharing, equal grouping, or repeated subtraction. Discuss each of the three representations of division.)
5. Have students make up their own division sentences in which a whole number is divided by a fraction. Have them model this division using all three meanings of division.

   Note: Students may need help coming up with division sentences that will divide well by a whole number. To help students recognize the pattern and generalize a rule, you will want to help students select numbers that work well together.
6. Discuss the various models with the class, encouraging a variety of responses:
   - Which model is best?
   - Which model is most clear?
   - Which model is easiest to understand?
7. Ask the recorder to share the division sentences and answers that he or she has been recording.
8. Have the class discuss any patterns they see. Ask students whether they can come up with a rule for dividing fractions and whole numbers based on what they have been doing.
9. Ask students to answer the following problem in their math journals:
   Jared has $\frac{3}{4}$ of a bag of sunflower seeds. He is sharing the bag with four of his friends. Approximately what fraction of the bag of sunflower seeds will each person get, assuming they all receive the same amount? (Students should be able to approximate the solution using models or pictures.)

Observation Checklist

☑ Observe students' responses to determine whether they can do the following:
- ❒ Connect the concept of dividing whole numbers to that of fractions.
- ❒ Represent division in a variety of ways.
- ❒ Divide proper fractions by whole numbers concretely, pictorially, and symbolically.
- ❒ Communicate mathematically.
- ❒ Recognize a pattern in order to generalize a rule.

Suggestions for Instruction

> - **Model multiplication of a positive fraction by a positive fraction, concretely or pictorially, and record the process.**
> - **Generalize and apply rules for multiplying and dividing positive fractions, including mixed numbers.**

Materials: Base-10 blocks, $8\frac{1}{2} \times 11$ paper, pencil crayons of different colours, BLM 5–8.5: Number Cards

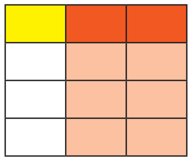
Organization: Whole class/individual/pairs

Procedure:

Part A: Proper Fractions

1. Have one student neatly record onto a transparency all multiplication sentences and answers talked about in class.
2. Tell students that one way to represent multiplication of whole numbers concretely or pictorially is to use the *area model*: length times width. Ask students to model $4 \bullet 6$ using base-10 blocks. Observe to ensure that the dimensions of the rectangle are 4×6 and that the area, or the space covered by the rectangle, is equal to the product of 4 and 6.
3. Provide students with paper, and ask them whether they can figure out a way to model $\frac{3}{4} \bullet \frac{1}{2}$, using the area model that was just reviewed. Give them the opportunity to explore their ideas before demonstrating how they can use paper folding to show multiplication of proper fractions.
4. Ask students to do the following:
   - Hold the paper in the landscape position and fold the paper to divide the paper into quarters.
   - Lightly shade in $\frac{3}{4}$ of the paper with a pencil crayon.
   - Turn the paper to the portrait position.
   - Fold the paper in half.
   - Lightly shade in $\frac{1}{2}$ of the paper with a pencil crayon of a different colour than the one used previously.
   - Into how many equal pieces is the paper folded now? (8)
   - Of those eight pieces, how many pieces have two colours? (3)
   - What fraction do the two coloured pieces represent? ($\frac{3}{4} \bullet \frac{1}{2}$ is $\frac{3}{8}$)
5. Ask students whether they can apply their understanding of how paper folding can be used to multiply fractions and solve the following using a diagram: $\frac{2}{3} \bullet \frac{1}{4}$.

6. Have students work through the following with the teacher:
   - Start by drawing a rectangle. Divide the rectangle into three equal parts and shade in two parts using a coloured pencil.



   - Divide the rectangle into four equal parts in the other direction (portrait). Shade in one of the four parts in a different colour.
   - Into how many equal parts is the rectangle divided? (12)
   - How many equal parts have two colours? (2)
   - What is that as a fraction? $\frac{2}{12}$ or $\frac{1}{6}$
   - So, $\frac{2}{3} \bullet \frac{1}{4} = \frac{2}{12}$ or $\frac{1}{6}$.
7. Discuss both the drawing method and the paper-folding method and have students decide which they like better.
8. Ask students to model $\frac{1}{5} \bullet \frac{3}{4}$ using paper folding or drawing.
9. Ask students to model $\frac{1}{2} \bullet \frac{7}{8}$ using paper folding or drawing.
10. Provide students with a set of number cards. Have them draw four cards to make two proper fractions. Have them multiply the fractions and then explain their method to a learning partner.
11. Ask the recorder to share the multiplication sentences and answers that he or she has been recording.
12. Have the class discuss any patterns they see. Ask students whether they can come up with a rule for multiplying proper fractions. Record this rule symbolically.

Observation Checklist

☑ Observe students' responses to determine whether they can do the following:
- ❒ Connect the concept of multiplying whole numbers to that of fractions.
- ❒ Represent multiplication in a variety of ways.
- ❒ Multiply proper fractions concretely, pictorially, and symbolically.
- ❒ Communicate mathematically.
- ❒ Recognize a pattern in order to generalize a rule.

Part B: Mixed Numbers

1. Use a series of steps similar to those outlined in Part A to have students develop strategies for multiplying mixed numbers.

Observation Checklist

☑ Observe students' responses to determine whether they can do the following:

- ❐ Connect the concept of multiplying whole numbers to that of fractions.
- ❐ Represent multiplication in a variety of ways.
- ❐ Multiply mixed numbers concretely, pictorially, and symbolically.
- ❐ Communicate mathematically.
- ❐ Recognize a pattern in order to generalize a rule.

Suggestions for Instruction

- **Generalize and apply rules for multiplying and dividing positive fractions, including mixed numbers.**
- **Identify and correct errors in the solution to a problem involving positive rational numbers.**

Materials: BLM 5–8.12: Fraction Bars or BLM 5–8.19: Double Number Line, BLM 5–8.11: Multiplication Table, BLM 8.N.6.4: Fraction Multiplication and Division, math journals

Organization: Whole class/individual

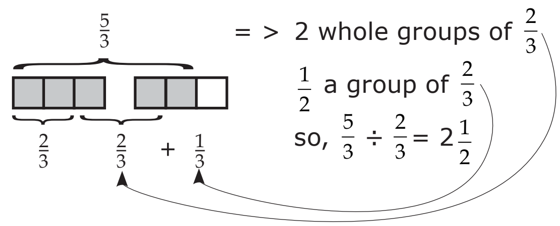
Procedure:

1. Tell students that by the end of this lesson, they will be able to determine and apply the rules for multiplying and dividing fractions, including mixed numbers.
2. Review division with students. Start with a whole number example (e.g., for 12 ÷ 3, think, how many groups of 3 go into 12?). Questions like this should put students in the mind frame that they are dividing a number into groups of a particular number.

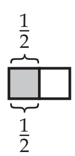
3. Students will need to use fraction bars to answer the following division questions:

	Division
1.	$\frac{1}{2} \div \frac{1}{2} =$
2.	$\frac{3}{4} \div \frac{2}{4} =$
3.	$\frac{4}{5} \div \frac{3}{5} =$
4.	$\frac{5}{6} \div \frac{4}{6} =$
5.	$\frac{5}{8} \div \frac{3}{8} =$
6.	$\frac{7}{10} \div \frac{2}{10} =$

Note: This concept may be very difficult for students to understand, so go through it slowly, one question at a time. The key is that the fraction becomes the portion of the divisor that is left over. (For example, in $\frac{5}{3} \div \frac{2}{3}$, $\frac{2}{3}$ can go into $\frac{5}{3}$ fully twice, $\frac{1}{2}$ of the $\frac{2}{3}$ is left over.)

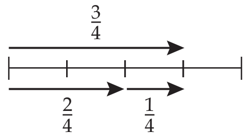


- Question #1 (need to have $\frac{1}{2}$ fraction bars):
  - Students will start out with $\frac{1}{2}$.
  - How many full groups of $\frac{1}{2}$ can you make? (1)
  - How many pieces are left over? (0)
  - So, the quotient is 1.



so, $\frac{1}{2} \div \frac{1}{2} = 1$

- Question #2 (need to have $\frac{1}{4}$ fraction bars):
  - Students will start out with $\frac{3}{4}$.
  - How many full groups of $\frac{2}{4}$ can be made from $\frac{3}{4}$? (1)
  - How many pieces are left over? (1)
  - So, one out of the two fraction bars that make $\frac{2}{4}$ is left over, or $\frac{1}{2}$ of the two fraction bars is left over. Therefore, the quotient is $1\frac{1}{2}$.

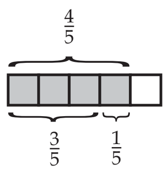


=> 1 whole group of $\frac{2}{4}$

$\frac{1}{2}$ of a group of $\frac{2}{4}$

so, $\frac{3}{4} \div \frac{2}{4} = 1\frac{1}{2}$

- Question #3 (need to have $\frac{1}{5}$ fraction bars):
  - Students will start out with $\frac{4}{5}$.
  - How many full groups of $\frac{3}{5}$ can you make? (1)
  - How many pieces are left over? (1)
  - So, one out of the three fractions bars that make $\frac{3}{5}$ is left over, or $\frac{1}{3}$ of the three fraction bars is left over. Therefore, the quotient is $1\frac{1}{3}$.

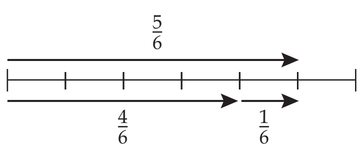


=> 1 whole group of $\frac{3}{5}$

$\frac{1}{3}$ of a group of $\frac{3}{5}$

so, $\frac{4}{5} \div \frac{3}{5} = 1\frac{1}{3}$

- Question #4 (need to have $\frac{1}{6}$ fraction bars):
  - Students will start out with $\frac{5}{6}$.
  - How many full groups of $\frac{4}{6}$ can you make? (1)
  - How many pieces are left over? (1)
  - So, one out of the four fraction bars that make $\frac{4}{6}$ is left over, or $\frac{1}{4}$ of the four fraction bars is left over. Therefore, the quotient is $1\frac{1}{4}$.

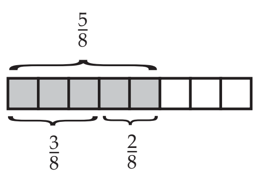


=> 1 whole group of $\frac{4}{6}$

$\frac{1}{4}$ of a group of $\frac{4}{6}$

so, $\frac{5}{6} \div \frac{4}{6} = 1\frac{1}{4}$

- Question #5 (need to have $\frac{1}{8}$ fraction bars):
  - Students will start out with $\frac{5}{8}$.
  - How many full groups of $\frac{3}{8}$ can you make? (1)
  - How many pieces are left over? (2)
  - So, two out of the three fraction bars that make $\frac{3}{8}$ are left over, or $\frac{2}{3}$ of the three fraction bars is left over. Therefore, the quotient is $1\frac{2}{3}$.

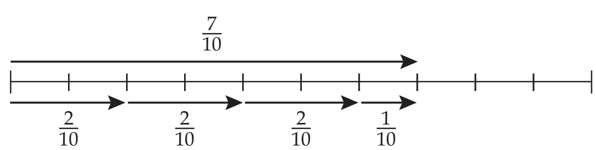


=> 1 whole group of $\frac{3}{8}$

$\frac{2}{3}$ of a group of $\frac{3}{8}$

so, $\frac{5}{8} \div \frac{3}{8} = 1\frac{2}{3}$

- Question #6 (need to have $\frac{1}{10}$ fraction bars):
  - Students will start out with $\frac{7}{10}$.
  - How many full groups of $\frac{2}{10}$ can you make? (3)
  - How many pieces are left over? (1)
  - So, one out of the two fraction bars that make $\frac{2}{10}$ is left over, or $\frac{1}{2}$ of the two fraction bars is left over. Therefore, the quotient is $3\frac{1}{2}$.



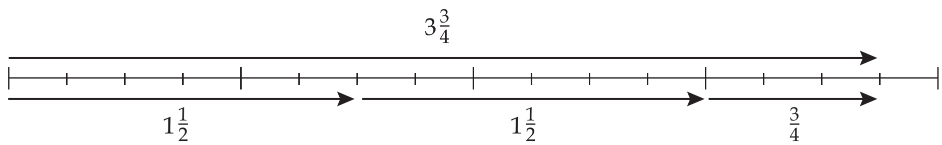
=> 3 whole groups of $\frac{2}{10}$

$\frac{1}{2}$ of a group of $\frac{2}{10}$

so, $\frac{7}{10} \div \frac{2}{10} = 3\frac{1}{2}$

4. Once students have solved the division questions, look at the multiplication questions. Students should already be familiar with a method for multiplication of fractions—numerator times numerator, denominator times denominator. Find the products from BLM 8.N.6.4: Fraction Multiplication and Division and have students draw a conclusion.
5. Ask students whether they can see a connection between the division statements and the multiplication statements. Hopefully, they will notice the method that if you multiply by the reciprocal, you end up with the same result as when you divide.

6. Ask students how that method can be applied to mixed numbers. (You need to change the mixed numbers to improper fractions and multiply by the reciprocal.)
7. Ask students to respond to the following question in their math journals:
   Jenna showed the following when calculating $3\frac{3}{4} \div 1\frac{1}{2}$.



$$3\frac{3}{4} \div 1\frac{1}{2} = 2\frac{3}{4}$$

   Do you agree with Jenna's thought process? Explain your response using words, symbols, and diagrams.

Observation Checklist

☑ Observe students' responses to determine whether they can do the following:

- ❒ Multiply proper fractions concretely.
- ❒ Multiply proper fractions pictorially.
- ❒ Multiply proper fractions symbolically.
- ❒ Multiply mixed numbers/improper fractions symbolically.
- ❒ Divide proper fractions concretely.
- ❒ Divide proper fractions pictorially.
- ❒ Divide proper fractions symbolically.
- ❒ Divide mixed numbers/improper fractions symbolically.

Suggestions for Instruction

- **Generalize and apply rules for multiplying and dividing positive fractions, including mixed numbers.**

Materials: BLM 8.N.6.5: Multiplying and Dividing Proper Fractions, Improper Fractions, and Mixed Numbers

Organization: Individual

Procedure:

1. Have students, individually, complete BLM 8.N.6.5: Multiplying and Dividing Proper Fractions, Improper Fractions, and Mixed Numbers.

Observation Checklist

☑ Observe students' responses to determine whether they can do the following:

- ❒ Multiply proper fractions symbolically.
- ❒ Multiply improper fractions symbolically.
- ❒ Multiply mixed numbers symbolically.
- ❒ Divide proper fractions symbolically.
- ❒ Divide improper fractions symbolically.
- ❒ Divide mixed numbers symbolically.

Suggestions for Instruction

- **Identify the operation(s) required to solve a problem involving positive fractions.**
- **Provide a context involving the multiplying of two positive fractions.**
- **Provide a context involving the dividing of two positive fractions.**
- **Identify the operation(s) required to solve a problem involving positive rational numbers.**
- **Determine the reasonableness of an answer to a problem involving positive rational numbers.**

Materials: BLM 8.N.6.6: Fraction Operations, chart paper, markers, math journals

Organization: Small group/whole class/individual

Procedure:

1. Have students form small groups, and provide each group with a copy of BLM 8.N.6.6: Fraction Operations. Explain that each group must
   - decide, as a group, what operation would be required for each problem and explain their reasoning
   - solve each problem
   - determine whether the solution seems reasonable and explain why they think their answer is reasonable
   - record their work on chart paper
2. Have groups present their work to the class, identifying various strategies they used to solve the problems. Observe whether there is a consensus as to which operation was used for each problem.
3. Provide opportunities for other students to add to and ask questions of the presenting groups.
4. Ask groups to create their own scenarios in which the multiplication and/or division of fractions is needed.
5. Ask each student to select two problems to solve, one that requires multiplication and one that requires division.
6. Have students solve these questions in their math journals, showing their work and discussing the reasonableness of their answers.

Observation Checklist

☑ Observe students' responses to determine whether they can do the following:

- ❐ Solve problems involving multiplication of fractions.
- ❐ Solve problems involving the division of fractions.
- ❐ Correctly identify the operation needed to solve a question involving fractions.
- ❐ Use reasoning to determine the reasonableness of an answer.
- ❐ Use mental mathematics and estimation during calculations.
- ❐ Provide a context involving the multiplication of two positive fractions.
- ❐ Provide a context involving the division of two positive fractions.

NOTES

Number—8.N.7

Enduring Understandings:

The principles of operations used with whole numbers also apply to operations with decimals, fractions, and integers.

A positive integer and a negative integer are opposites when they are the same distance from zero on a number line.

The sum of two opposite numbers is 0.

General Learning Outcome:

Develop number sense.

Specific Learning Outcome(s):	Achievement Indicators:
8.N.7 Demonstrate an understanding of multiplication and division of integers, concretely, pictorially, and symbolically. [C, CN, PS, R, V]	➛ Identify the operation(s) required to solve a problem involving integers. ➛ Provide a context that requires multiplying two integers. ➛ Provide a context that requires dividing two integers. ➛ Model the process of multiplying two integers using concrete materials or pictorial representations, and record the process. ➛ Model the process of dividing an integer by an integer using concrete materials or pictorial representations, and record the process. ➛ Generalize and apply a rule for determining the sign of the product or quotient of integers. ➛ Solve a problem involving integers, taking into consideration order of operations.

Prior Knowledge

Students may have had experience with the following:

- Demonstrating an understanding of integers, concretely, pictorially, and symbolically
- Explaining and applying the order of operations, excluding exponents (limited to whole numbers)
- Demonstrating an understanding of addition and subtraction of integers, concretely, pictorially, and symbolically

Background Information

Multiplication and Division of Integers

When multiplying two numbers to get a product, the numbers being multiplied are called *factors*. For example, in 4 • 2 = 8, 4 and 2 are factors. Regardless of what order the factors are written, the product is the same (4 • 2 = 8 or 2 • 4 = 8). This is called the *commutative property*.

commutative property

A number property that states that an operation (addition or multiplication) is unaffected by the order in which the terms are added or multiplied.

Examples:

Addition

The sum remains the same (e.g., 2 + 3.5 = 3.5 + 2).

Multiplication

The product remains the same (e.g., 3 • 5 = 5 • 3).

Applying the commutative property to integers:

- (+4) • (–2) = (–8)

  This can be described as having four groups of –2, or (–2) + (–2) + (–2) + (–2).
- (–2) • (+4) = (–8)

  This can be described as having the negative of two groups of 4, or –[(+4) + (+4)].

Both (+4) • (–2) and (–2) • (+4) have the same value.

An *opposite integer* is an integer that, when added to another, creates a sum of zero. This is called the *zero principle*. *Opposite integers* are integers that are of equal distance from zero on a number line. Opposite integers are sometimes referred to as *zero pairs*. For example, –3 and +3 are opposite integers. You can also think of the multiplication scenario of –2 • 4 as the opposite of 2 times 4, and since 2 times 4 is 8, the opposite of that must be –8.

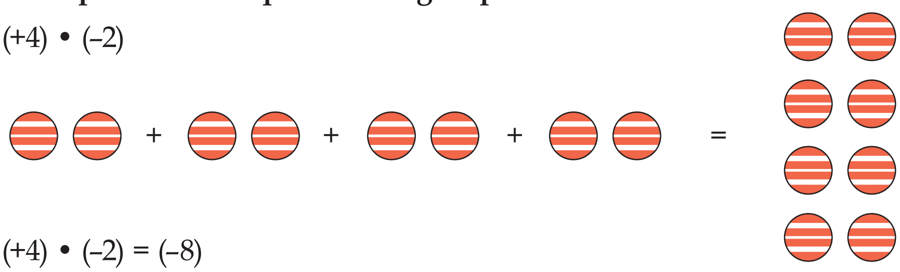
Integer disks or number lines can be used to explore and model the product or quotient of integers.

Multiplication and Division Examples

In the following examples, the blue (solid) bingo chips represent positive integers and the red (striped) bingo chips represent negative integers. A zero pair is the pair made by –1 and +1
(and) since its sum is zero.

- **Multiplication as equal sets or groups:**

(+4) • (–2)

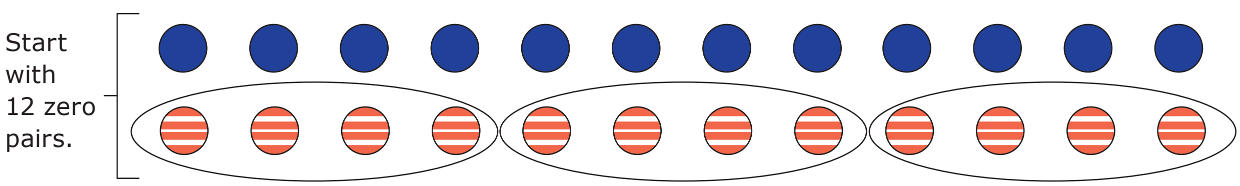


(+4) • (–2) = (–8)

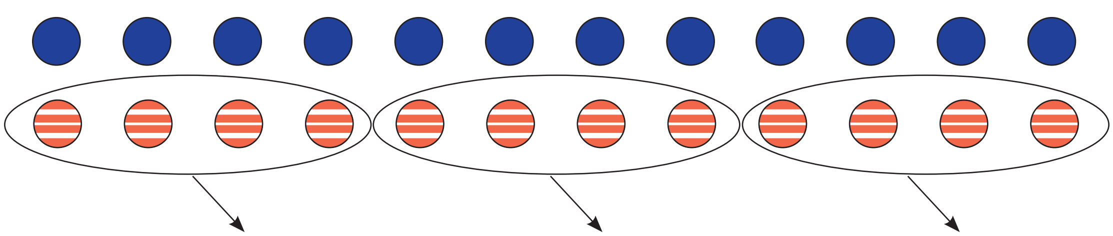
(–3) • (–4)

This is like saying three groups of –4 are being removed.

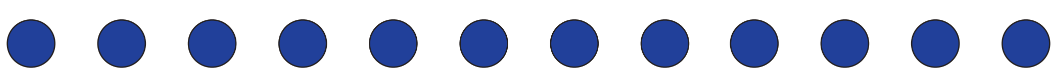
In order to do this, you need to start with zero, yet you must have enough zeros (or zero pairs) so that you can take away three groups of –4.



Take away three groups of –4.



You are left with +12.



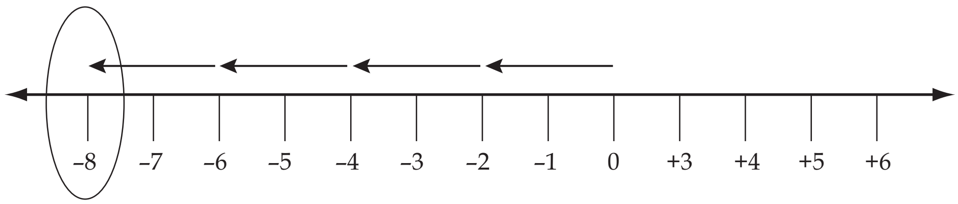
(–3) • (–4) = (+12)

(–3) • (–4) could also be thought of as the opposite of three groups of –4.

- **Multiplication as repeated addition:**

  (+4) • (–2)

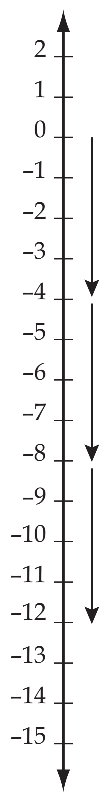
  –2 + –2 + –2 + –2



  (+4) • (–2) = (–8)

  (–3) • (–4)

This is like saying, the opposite of three groups of –4, and since three groups of –4 is –12, the opposite of that would be +12.



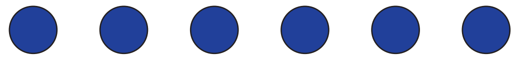
3 • –4 is –12

so, –3 • –4 is +12

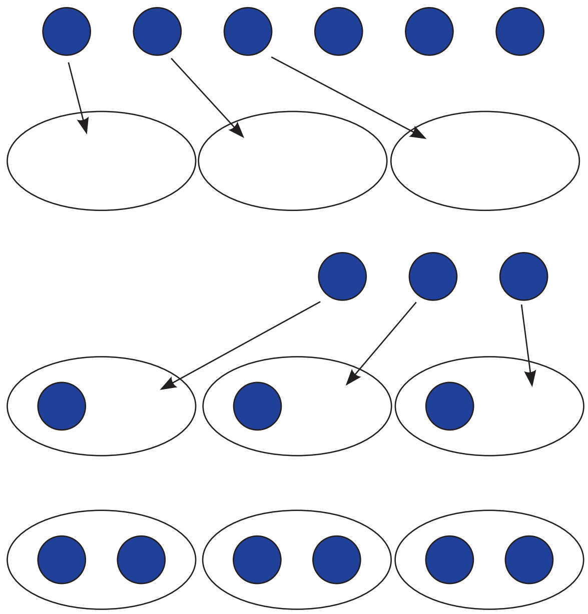
(–3) • (–4) = (+12)

- **Division as equal sharing:**

  (+6) ÷ (+3)



  This can be interpreted as +6 shared equally among +3 groups.

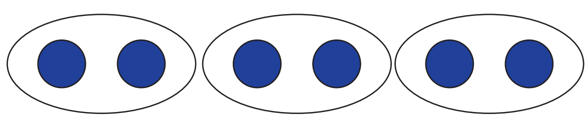


  (+6) ÷ (+3) = (+2)

- **Division as equal grouping:**

  (+6) ÷ (+3)

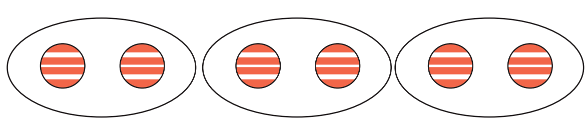
  This can be interpreted as 6 divided into three equal groups. How much is in each group?



  (+6) ÷ (+3) = (+2)

  (–6) ÷ (+3)

  This can be interpreted as –6 divided into three equal groups. How much is in each group?



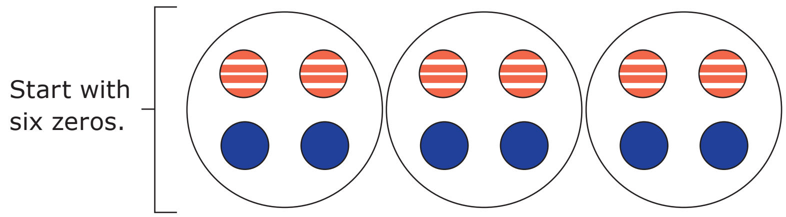
  (–6) ÷ (+3) = (–2)

- **Division as repeated subtraction:**

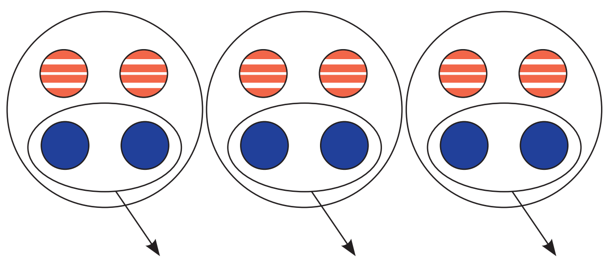
  (+6) ÷ (–3)

  Remember that when you multiplied a negative number by a negative number, it was the same as repeated subtraction. Division of negative numbers is similar. When you are dividing a positive number by a negative number, you will be removing the opposite number from the group, and determining what is left. Once again, you must start with zero before you can remove any integers, and then determine what is left in each group.

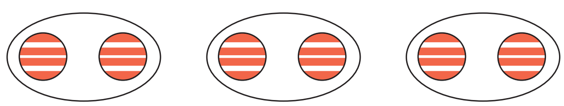
  For example, with +6 ÷ –3, you are dividing the six zeros into three groups.



  Each group has two zeros. From each zero group, you are removing groups of positive numbers.



  The result is that each group has –2 left.

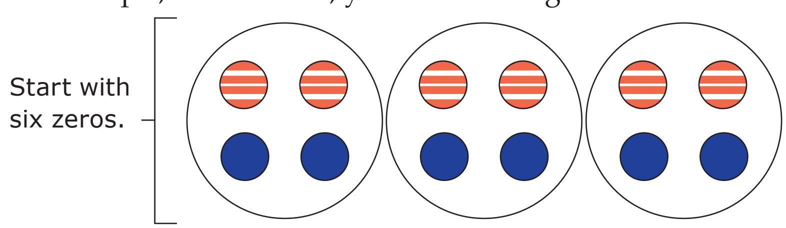


  (+6) ÷ (–3) = (–2)

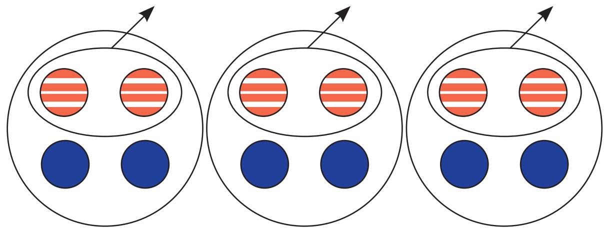
  (–6) ÷ (–3)

  When you are dividing a negative number by a negative number, you are removing groups of negative numbers from the zeros and will end up with positive groups of numbers.

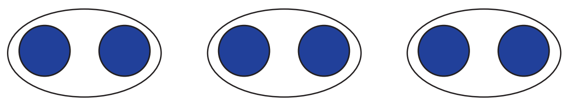
  For example, with –6 ÷ –3, you are dividing six zeros into three groups.



Each group has two zeros. From each zero group, you are removing groups of negative numbers.



The result is that each group has +2 left.



$(-6) \div (-3) = (+2)$

- **Division using fact families:**

  You can also think of division with integers with respect to their fact families. A *fact family* is anything that is true of a set of numbers.

  Example:

  $3 \bullet 5 = 15; 5 \bullet 3 = 15; 15 \div 5 = 3; 15 \div 3 = 5$

  Example:

  $(-6) \div (-3)$

  So, if students have an easier time understanding why multiplying integers makes sense, work with that knowledge and students' knowledge of fact families to build a generalization.

  $(-6) \div (-3) = ?$

  Students may think: $(-3) \bullet ? = (-6)$

  $(-3) \bullet (2) = -6; (2) \bullet (-3) = -6; (-6) \div (2) = -3; (-6) \div (-3) = (2)$

  Therefore, $(-6) \div (-3) = 2$

Mathematical Language

commutative property
integer
opposite integer
order of operations
zero pair
zero principle

Assessing Prior Knowledge

Materials: Positive and negative integer disks or bingo chips, BLM 8.N.7.1: Integer Pre-Assessment

Organization: Individual

Procedure:

1. Tell students that they will be extending their understanding of integers; however, you first need to determine what they already know about integers.
2. Provide each student with a group of positive and negative integer disks, as well as a copy of BLM 8.N.7.1: Integer Pre-Assessment. Ask students to complete this BLM.

Observation Checklist

☑ Observe students' responses to determine whether they can do the following:

- ❒ Demonstrate that zeros do not change the value of integers.
- ❒ Add integers with like signs.
- ❒ Add integers with opposite signs.
- ❒ Subtract integers with like signs when the first term is larger than the second term.
- ❒ Subtract integers with like signs when the first term is smaller than the second term.
- ❒ Subtract integers with opposite signs.

Suggestions for Instruction

- **Model the process of multiplying two integers using concrete materials or pictorial representations, and record the process.**
- **Generalize and apply a rule for determining the sign of the product or quotient of integers.**

Materials: Positive and negative integer disks, math journals

Organization: Whole class/individual/pairs

Procedure:

1. Review the concept of multiplication by asking students how they can represent 4 • 2. Include a discussion of commutative property, multiplication as repeated addition, and multiplication as groups/sets of. Include representations using integer disks and number lines.
2. Ask students what kind of numbers 4 and 2 are from the previous question. Discuss that they are positive integers.
3. Ask students whether they can apply their understanding of multiplication of positive integers to model (+5) • (–3). Think five groups of –3 or (–3) + (–3) + (–3) + (–3) + (–3).
4. Ask students whether they can apply their understanding of multiplication of positive integers to model (–5) • (+3). Discuss their thinking and bring in the definition of commutative property, so that the question can be understood as (+3) • (–5) or three groups of –5.
5. Have students use their integer disks to model the following: (+5) • (+2), (+3) • (–4), (–2) • (+6). Ask students to look at the signs of the terms and then look at the sign of the products. What observations can students make?
6. Ask students to use integer disks to model the following: (–4) • (–3). Discuss their thinking. Record students' responses on the whiteboard. With each response, discuss the pros and cons to determine ways to model (–4) • (–3).
7. Ask students to model the following: (+6) • (+2), (–4) • (+3), (+2) • (–2), (–3) • (–5). Have them record their process in their math journals using pictures, words, symbols, and numbers. Ask them to generalize a rule for determining the sign of the product of two integers.

Note: A possible method is to look at multiplying negative integers as repeated subtraction. Start with 12 zeros to model this statement and remove three groups of −4 from the zeros. What is left? Have students look at the signs of the terms and then look at the sign of the quotient. What do they notice?

8. Have students create four new multiplication sentences similar to those above, and exchange sentences with a partner. Have students determine the products symbolically.

Observation Checklist

☑ Observe students' responses to determine whether they can do the following:

- ❒ Use a model to show the multiplication of integers.
- ❒ Record the process of multiplication of integers.
- ❒ Generalize a rule for determining the sign of the product of integers.
- ❒ Apply a rule to determine the sign of the product of integers.

Suggestions for Instruction

- **Provide a context that requires multiplying two integers.**
- **Solve a problem involving integers, taking into consideration order of operations.**

Materials: BLM 8.N.7.2: Solving Problems with Integers (A), chart paper, math journals

Organization: Small group/whole class/individual

Procedure:

1. Have students form small groups, and provide each group with a copy of BLM 8.N.7.2: Solving Problems with Integers (A).
2. Ask the groups to record their responses to the problems on chart paper and be prepared to share their solutions with the rest of the class. In their presentations, they must be able to explain why they chose to solve the problems the way they did.
3. As a class, discuss the solutions that the groups present. Allow opportunities to discuss different ways of solving the problems.
4. Ask students to create and solve their own problem requiring multiplication of integers. Have them record their work in their math journals.

Observation Checklist

☑ Observe students' responses to determine whether they can do the following:

- ❒ Describe a real-world scenario in which the multiplication of integers is required.
- ❒ Solve problems requiring the multiplication of integers.
- ❒ Communicate problem-solving strategies.

Suggestions for Instruction

- **Model the process of dividing an integer by an integer using concrete materials or pictorial representations, and record the process.**
- **Generalize and apply a rule for determining the sign of the product or quotient of integers.**

Materials: Positive and negative integer disks, math journals

Organization: Whole class/individual

Procedure:

1. Review the concept of division by asking students how they can represent (+24) ÷ (+4).
2. Record all the different ways students represent 24 ÷ 4. Make sure that using integer disks is one way in which students model the division.
3. Ask students what kind of numbers 24 and 4 are from the previous question. In the discussion, indicate that they are positive integers.
4. Ask students to predict, using their knowledge of the multiplication of integers, what some rules might be for division of integers. Tell students that you will work with them to see whether their rules work.
5. Ask students whether they can apply their understanding of the division of positive integers to model (–15) ÷ (+3) using integer disks. Discuss their thinking, and explain that the model can be interpreted as the value of each group when –15 is divided into three groups.

6. Ask students how the following can be interpreted: (+15) ÷ (–3). In the discussion, explain that dividing by a negative number is like subtracting from a group. (Refer to the explanation in the Background Information.) Demonstrate how integer disks can be used to model division by starting with zero, removing a number from the group, and examining the result. Ask students to look at the signs of the terms and then look at the sign of the quotients. What observations can students make?
7. Set up the following and ask students to make connections.

Multiplication	Division
(+4) • (+2) = (+8)	(+8) ÷ (+2) = (+4)
(+4) • (–2) = (–8)	(+8) ÷ (–2) = (–4)
(–4) • (+2) = (–8)	(–8) ÷ (+2) = (–4)
(–4) • (–2) = (+8)	(–8) ÷ (–2) = (+4)

See whether students can come up with the following generalizations:

Multiplication	Division
Positive times a positive = positive	Positive divided by a positive = positive
Positive times a negative = negative	Positive divided by a negative = negative
Negative times a positive = negative	Negative divided by a positive = negative
Negative times a negative = positive	Negative divided by a negative = positive

8. Ask students to model the following: (+9) ÷ (+3), (+12) ÷ (–4), (–10) ÷ (+2), (–15) ÷ (–5). Have them record their process in their math journals. Ask them to write a rule for determining the sign of the product and quotient of integers.

Observation Checklist

☑ Observe students' responses to determine whether they can do the following:

- ❒ Use integer disks to model division of integers.
- ❒ Record the process of division of integers pictorially and symbolically.
- ❒ Generalize a rule for determining the sign of the quotient of integers.
- ❒ Apply a rule to determine the sign of the quotient of integers.

Suggestions for Instruction

- **Provide a context that requires dividing two integers.**
- **Solve a problem involving integers, taking into consideration order of operations**

Materials: BLM 8.N.7.3: Solving Problems with Integers (B), chart paper, math journals

Organization: Small group/whole class/individual

Procedure:

1. Have students form small groups, and provide each group with a copy of BLM 8.N.7.3: Solving Problems with Integers (B).
2. Ask the groups to record their responses to the problems on chart paper and be prepared to share their solutions with the rest of the class. In their presentations, they must be able to explain why they chose to solve the problems the way they did.
3. As a class, discuss the solutions that the groups present. Allow opportunities to discuss different ways of solving the problems.
4. Ask students to create and solve their own problem requiring the division of integers. Have them record their work in their math journals.

Observation Checklist

☑ Observe students' responses to determine whether they can do the following:

- ❒ Describe a real-world scenario in which the division of integers is required.
- ❒ Solve problems requiring the division of integers.
- ❒ Communicate problem-solving strategies.

- **Solve a problem involving integers, taking into consideration order of operations.**
- **Identify the operation(s) required to solve a problem involving integers.**

Materials: BLM 8.N.7.4: Solving Problems with Integers (C), chart paper, math journals

Organization: Small group/whole class/individual

Procedure:

1. Have students form small groups, and provide each group with a copy of BLM 8.N.7.4: Solving Problems with Integers (C).
2. Ask the groups to record their responses to the problems on chart paper and be prepared to share their solutions with the rest of the class. In their presentations, they must be able to explain why they chose to solve the problems the way they did.
3. As a class, discuss the solutions that the groups present. Allow opportunities to discuss different ways of solving the problems.
4. Ask students to create and solve their own problem requiring the order of operations with integers. Have them record their work in their math journals.

Observation Checklist

☑ Observe students' responses to determine whether they can do the following:

- ❒ Describe a real-world scenario in which the order of operations with integers is required.
- ❒ Solve problems requiring the order of operations with integers.
- ❒ Identify the operation(s) needed to solve an integer problem.
- ❒ Communicate mathematically to solve problems.

- **Generalize and apply a rule for determining the sign of the product or quotient of integers.**

Materials: Deck of cards—Ace = 1, Jack = 11, Queen = 12, King = 0, other cards at face value (assign black cards as positive and red cards as negative)

Organization: Pairs

Procedure:

1. Pair up students in the class, with one student on each side of a desk. Arrange desks in a circle with one student on the inside of each desk and one student on the outside. Tell students they will play an integer multiplication game.
2. Demonstrate how to play the integer multiplication game.
   - Two players divide cards evenly between themselves.
   - The two players each turn over a card simultaneously and multiply the face value of the two cards. The fastest responder with the correct answer wins the hand. In the event of a tie, students turn over two more cards until one player says the correct answer out loud.
   - After about five minutes of play, the students in the inner circle rotate to the right to challenge a new set of students. Students can also challenge individuals in the room who they believe are at the same level as they are.

Variation: Students could play a variation of the game.

- Two players divide cards evenly between themselves.
- Students each turn over two cards simultaneously. Each student multiplies his or her own cards and the person with the greatest product collects all four cards. In the event of a tie (same answer), players turn over two more cards and multiply them.

Observation Checklist

☑ Observe students' responses to determine whether they can do the following:

- ❒ Multiply positive and negative integers.
- ❒ Determine the greatest value of products.

- **Generalize and apply a rule for determining the sign of the product or quotient of integers.**
- **Identify the operation(s) required to solve a problem involving integers.**

Materials: Ten-sided number cube and integer disks (1 each per group), BLM 8.N.7.5: Number Line Race

Organization: Whole class/pairs

Procedure:

1. Have students form pairs, and provide each pair with a number cube and integer disk, as well as a copy of BLM 8.N.7.5: Number Line Race.
2. Tell students that they will be playing a game called *Number Line Race*. The object of the game is to cross out all numbers on a number line before their opponent does.
3. Demonstrate to the class how to play the game.
   - Player A rolls the number cube and flips the integer disk, and player B records the number in the Numbers Rolled column of his or her own chart (e.g., a 7 on the number cube and the chip lying red face up would be –7).
   - Repeat two more times, until the pair has recorded three integers.
   - Players A and B work individually for one minute using two or three of the integers and the order of operations to make as many integers as they can, recording all work in the Numbers Found column of their own charts.
   - Each player will cross out the numbers found on his or her own number line.
   - Play continues until one player has crossed out all numbers on the number line.
4. Have students play the game.

Observation Checklist

☑ Observe students' responses to determine whether they can do the following:

- ❒ Apply a rule for determining the sign of the product or quotient of integers.
- ❒ Identify the order of operations needed to make the needed numbers on their number line.
- ❒ Apply mental mathematics and reasoning skills while playing the game.

Grade 8 Mathematics

Patterns and Relations

Patterns and Relations (Patterns)—8.PR.1

Enduring Understandings:

Words, tables, graphs, expressions, and equations are different representations of the same pattern.

Variables are used to describe mathematical relationships.

General Learning Outcome:

Use patterns to describe the world and solve problems.

Specific Learning Outcome(s):	Achievement Indicators:
8.PR.1 Graph and analyze two-variable linear relations. [C, ME, PS, R, T, V]	➣ Determine the missing value in an ordered pair for an equation of a linear relation. ➣ Create a table of values for the equation of a linear relation. ➣ Construct a graph from the equation of a linear relation (limited to discrete data). ➣ Describe the relationship between the variables of a graph.

Prior Knowledge

Students may have had experience with the following:

- Representing and describing patterns and relationships using charts and tables to solve problems
- Identifying and explaining mathematical relationships using charts and diagrams to solve problems
- Determining the pattern rule to make predictions about subsequent elements
- Representing and describing patterns and relationships using graphs and tables
- Demonstrating an understanding of oral and written patterns and their corresponding relations
- Constructing a table of values from a relation, graphing the table of values, and analyzing the graph to draw conclusions and solve problems
- Identifying and plotting points in the four quadrants of a Cartesian plane using ordered pairs

Background Information

Increasing/Decreasing Linear Patterns

Although there are different types of patterns, the learning experiences that follow focus on *increasing/decreasing **linear** patterns*. The elements that make up these patterns are called *terms*. Each term builds on the previous term. Consequently, these patterns are often referred to as *growing patterns*. For example, 2, 4, 6, 8, 10 . . . and 0, 3, 6, 9, 12 . . . are two common increasing linear patterns.

Using a table to model an increasing/decreasing pattern can help students organize their thinking. It can also help them generalize the patterns symbolically. There are two types of generalizations that can be made: recursive and explicit.

- A *recursive generalization* tells how to find the value of a term given the value of the preceding term.
- An *explicit generalization* expresses the relationship between the value of the term and the term number.

Example:

Consider this pattern: ♥ ♥♥♥ ♥♥♥♥♥ ♥♥♥♥♥♥♥ ♥♥♥♥♥♥♥♥♥

The pattern can be organized into a table like this:

Term	1	2	3	4	5
Term Value	1	3	5	7	9

The recursive generalization that describes this pattern is $n + 2$ (where n is the previous term value), since the value of each term is two more than the preceding term. If the pattern were continued, the value of the sixth term would be 11, since 9 + 2 = 11. However, to find the value of the 100th term, you would need to find the value of each of the 99 preceding terms.

It is easier to predict the value of subsequent terms with an explicit generalization. Notice that in the above pattern, if you double a term number and subtract 1 ($2n – 1$), you get the value of the term (where n is the term number).

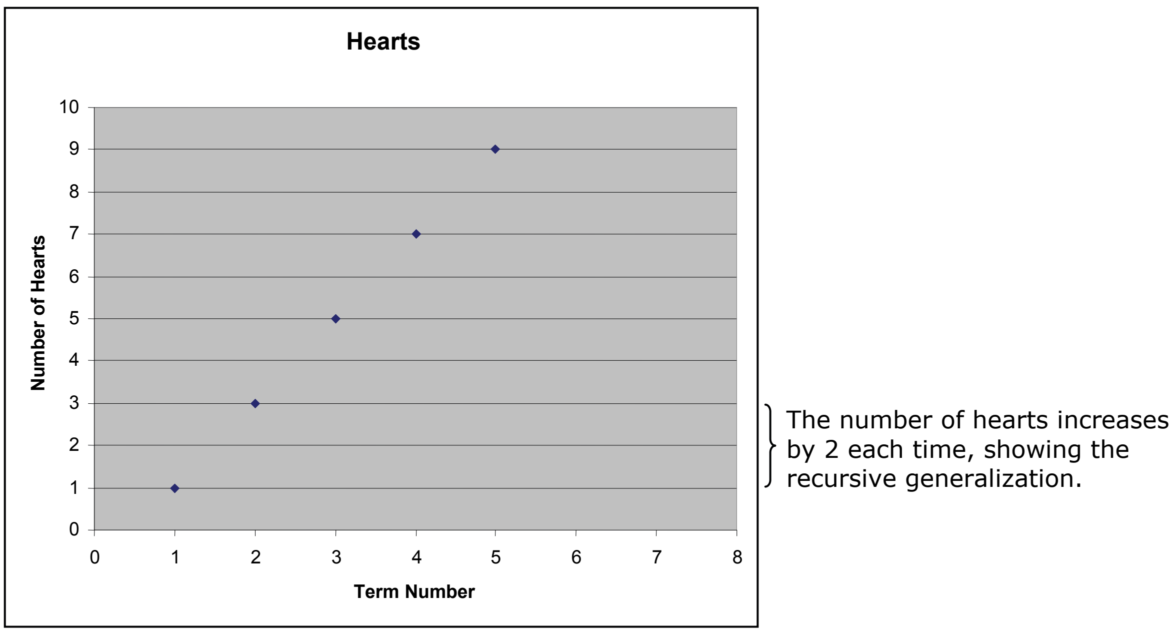
The value of the third term is 2 • 3 – 1 = 5, and the value of the fifth term is 2 • 5 – 1 = 9. Thus, the explicit generalization that describes the pattern is $2n - 1$. If the pattern were continued, the value of the 100th term would be 199, since 2 • 100 – 1 = 199.

When helping students to recognize patterns, it is important to remember that they may not see the patterns in the same way you do. Therefore, ask students to explain their thinking. Having students describe their reasoning can also help them realize that often there is more than one way to look at a pattern.

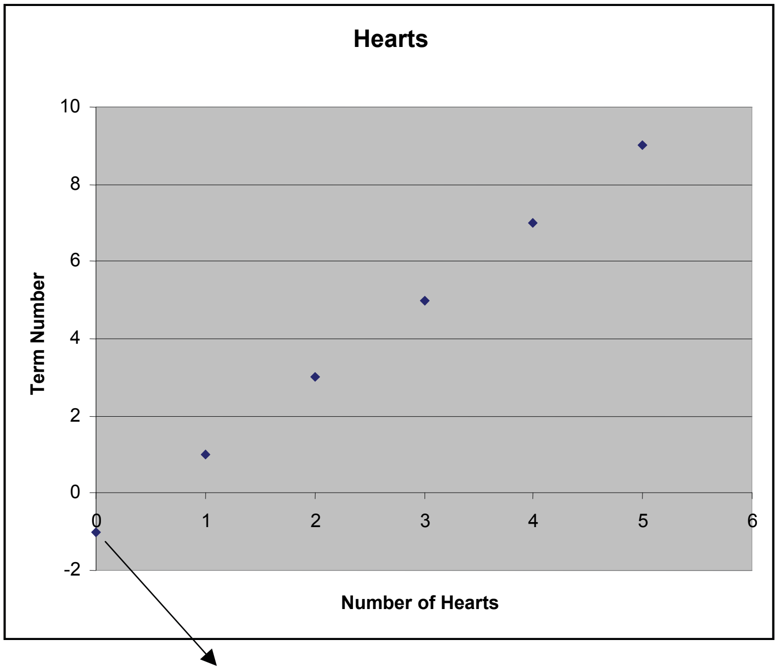
Graphing linear relationships can often help students recognize both the recursive and explicit generalizations.

Example:

The graph for the above pattern would look like this:



The explicit generalization of $2n - 1$ can be seen in the graph:



The 2 in the equation comes from an increase in 2 hearts for every 1 term number (referred to as *slope*).

The −1 in the equation comes from extending the visual line until it meets the *y*-axis (referred to as the *y*-intercept).

Note: Students will not formally encounter the slope and the *y*-intercept until Senior Years mathematics. It is important, however, for students to analyze linear relationships expressed graphically. If students make these generalizations, provide them with the correct terminology.

The data in the above graphs are displayed as *discrete data*. This occurs when it would not make sense to plot points in between graphed values (e.g., the term number 1.5 does not exist). If you were to connect the points, it would be referred to as *continuous data* (e.g., the graph of the distance walked in an amount of time).

Mathematical Language

equation

expression

formula

linear relation

pattern

relation

table of values

variable

x-value

y-value

Learning Experiences

Assessing Prior Knowledge

Materials: BLM 8.PR.1.1: Patterns Pre-Assessment

Organization: Individual

Procedure:

1. Tell students that they will be extending their understanding of patterns over the next few lessons; however, you first need to determine what they already know about patterns.
2. Hand out copies of BLM 8.PR.1.1: Patterns Pre-Assessment.
3. Have students complete all sections of the BLM to the best of their ability.

Observation Checklist

☑ Observe students' responses to determine whether they can do the following:

- ❒ Extend a pattern pictorially.
- ❒ Describe a pattern in own words.
- ❒ Construct a table of values from a relation.
- ❒ Make predictions from a pattern.
- ❒ Graph relations.
- ❒ Write relations algebraically.

Suggestions for Instruction

- **Determine the missing value in an ordered pair for an equation of a linear relation.**

Materials: BLM 8.PR.1.2: Determine the Missing Values

Organization: Whole class/individual

Procedure:

1. Put the following chart on the whiteboard.

x	1		3	4		
y	0	2	4			

2. Ask students whether they can fill in the missing x and y values. Ask them how they determined the missing values.
3. Ask students to record the pattern as an equation. Record the following ordered pairs, one at a time, on the board: (8, y), (x, 20), (0, y), (x, –4). Ask students to determine the missing values for the pattern.
4. Discuss student responses. Record any suggestions students make and evaluate the various methods used to determine the missing values for the ordered pairs.
5. Have students individually complete BLM 8.PR.1.2: Determine the Missing Values.

Observation Checklist

☑ Observe students' responses to determine whether they can do the following:

- ❐ Determine the missing values in a table.
- ❐ Determine the missing value in an ordered pair for a linear equation.

Suggestions for Instruction

- **Determine the missing value in an ordered pair for an equation of a linear relation.**

Materials: BLM 8.PR.1.3: Break the Code, projector

Organization: Whole class/pairs

Procedure:

1. Display BLM 8.PR.1.3: Break the Code to the class (using a projector).
2. As a class, work through the ordered pairs to break the code.
3. Have students work in pairs, creating codes for one another and then exchanging them with their partners.

Observation Checklist

☑ Observe students' responses to determine whether they can do the following:

- ❐ Determine the missing value in an ordered pair for a linear equation.

Suggestions for Instruction

- **Create a table of values for the equation of a linear relation.**
- **Construct a graph from the equation of a linear relation (limited to discrete data).**

Materials: BLM 8.PR.1.4: Linear Relations

Organization: Whole class/individual

Procedure:

1. As a class, work through the following problems. Discuss how you select the numbers in the table of values. Model how you set up the tables and how you graph the data. Also, discuss the linear relations that are represented in the graphs.

   Example 1:

   - The circumference of a circle is approximately three times its diameter. The precise relationship can be expressed as $C = \pi d$, where C is the circumference, π is a constant of 3.14159 . . . , and d is the diameter.
   - Make a table of values to show the relationship.
   - Construct a graph from the ordered pairs you recorded from the equation.

   Example 2:

   - The following is a linear relation: $y = 3x + 1$.
   - Make a table of values to show this relationship.
   - Construct a graph from the ordered pairs.

2. Discuss the graphs with students to ensure they understand correct graphing technique, including selecting a title, labelling axes, recording dependent variable on the *y* axis, recording independent variable on the *x* axis, spacing numbers appropriately, and so on.
3. Hand out copies of BLM 8.PR.1.4: Linear Relations. Have students complete the learning activity individually.

Observation Checklist

☑ Observe students' responses to determine whether they can do the following:

- ❒ Independently create a table of values for the equation of a linear relation.
- ❒ Construct a graph from the equation of a linear relation.

Suggestions for Instruction

- **Describe the relationship between the variables of a graph.**

Materials: BLM 8.PR.1.5: Graphs

Organization: Individual/pairs/whole class

Procedure:

1. Hand out copies of BLM 8.PR.1.5: Graphs.
2. Ask students to create a T-table or a T-chart to represent the *x* and *y* values found on the graphs.
3. Ask students to describe the relationship between the *x* and *y* values found on the graphs and in the T-chart using a formula.
4. Have students join a partner and share the relationships they each identified. Extend the discussion by having students identify other variables that could result in a graph that looks like the ones on BLM PR.1.5.
5. Discuss the identified relationships as a class.
6. Have students find a graph (e.g., in newspapers, magazines) and see whether they can identify a linear relationship in the graph.
7. Have students individually draw a graph that reflects a linear relationship, and then exchange it with their respective partners, who must identify the linear relationship between the variables on the graph.

Observation Checklist

☑ Observe students' responses to determine whether they can do the following:

- ❒ Communicate mathematically.
- ❒ Visualize a mathematical relationship that will create a linear relation.
- ❒ Describe the relationship between variables of a graph.

NOTES

Patterns and Relations (Variables and Equations)—8.PR.2

Enduring Understandings:

The principles of operations used with whole numbers also apply to operations with decimals, fractions, and integers.

Number sense and mental mathematics strategies are used to estimate answers and lead to flexible thinking.

Preservation of equality is used to solve equations.

General Learning Outcome:

Represent algebraic expressions in multiple ways.

Specific Learning Outcome(s):	Achievement Indicators:
8.PR.2 Model and solve problems using linear equations of the forms ▪ $ax = b$ ▪ $\frac{x}{a} = b, a \neq 0$ ▪ $ax + b = c$ ▪ $\frac{x}{a} + b = c, a \neq 0$ ▪ $a(x + b) = c$ concretely, pictorially, and symbolically, where *a*, *b*, and *c* are integers. [C, CN, PS, V]	➛ Model a problem with a linear equation, and solve the equation using concrete models. ➛ Verify the solution to a linear equation using a variety of methods, including concrete materials, diagrams, and substitution. ➛ Draw a visual representation of the steps used to solve a linear equation and record each step symbolically. ➛ Solve a linear equation symbolically. ➛ Identify and correct errors in an incorrect solution of a linear equation. ➛ Solve a linear equation by applying the distributive property (e.g., $2(x + 3) = 5$; $2x + 6 = 5$; . . .). ➛ Solve a problem using a linear equation, and record the process.

Prior Knowledge

Students may have had experience with the following:

- Applying mental mathematics strategies for multiplication, such as
  - annexing, then adding zeros
  - halving and doubling
  - using the distributive property
- Solving problems involving single-variable (expressed as symbols or letters), one-step equations with whole-number coefficients and whole-number solutions
- Demonstrating and explaining the meaning of preservation of equality, concretely, pictorially, and symbolically
- Demonstrating an understanding of preservation of equality by
  - modelling preservation of equality, concretely, pictorially, and symbolically
  - applying preservation of equality to solve equations
- Explaining the difference between an expression and an equation
- Evaluating an expression given the values of the variable(s)
- Modelling and solving problems that can be represented by one-step linear equations of the form $x + a = b$, concretely, pictorially, and symbolically, where a and b are integers
- Modelling and solving problems that can be represented by linear equations of the form:
  - $ax + b = c$
  - $ax = b$
  - $\frac{x}{a} = b, a \neq 0$

  concretely, pictorially, and symbolically, where a, b, and c are whole numbers

Related Knowledge

Students should be introduced to the following:

- Demonstrating an understanding of multiplying and dividing positive fractions and mixed numbers, concretely, pictorially, and symbolically
- Demonstrating an understanding of multiplication and division of integers, concretely, pictorially, and symbolically

Background Information

Linear Equations

Linear equations enable students to solve problems and make predictions based on a linear relationship. Understanding that linear equations form a balance will assist with understanding the relationships found in other mathematical concepts (e.g., Pythagorean theorem, area of a triangle, area of a circle).

The learning experiences related to specific learning outcome 8.PR.2 focus, in part, on translating word problems into equations and then solving them. This is not new to students. In the Early Years, students were introduced to whole-number operations through routine problems. At first, students solved these problems using concrete and pictorial representations. Later, they translated these problems into equations, often using an empty square to represent the unknown value.

Consequently, the learning experiences suggested for this learning outcome provide students with additional experience with solving routine problems. At the same time, students begin the transition to using letters to represent unknown quantities.

The suggested learning experiences also serve as an informal introduction to the terms *equation*, *mathematical expression*, and *variable*, which are defined as follows:

- An *equation* is a mathematical sentence stating that one or more quantities are equal. Equations that contain variables, such as $3 + x = 21$ and $2y + 3 = 15$, are sometimes referred to as *open sentences*, while equations that have no variables, such as $3 + 5 = 8$ and $24 \div 3 = 8$, are referred to as *closed sentences*.
- A *mathematical expression* comprises numbers, variables, and operation signs, but does not contain a relational symbol such as $=$, $\neq$, $<$, $>$, $\leq$, or $\geq$. For example, $6x + 3$ and $\frac{x}{4} - 8$ are mathematical expressions.
- A *variable* is a symbol for a number or a group of numbers in a mathematical expression or an equation.

In Grade 8, students are expected to solve two-step linear equations where coefficients and constants are integers, as well as apply the distributive property to equations in order to solve them.

Two-step linear equations have two operations within the expression and, therefore, the most efficient method to solve the problem involves two steps.

Example:

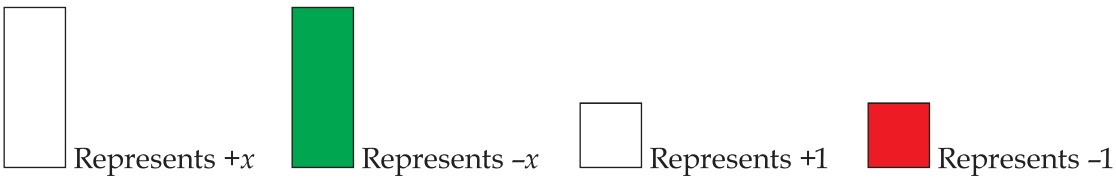
$2x + 6 = 5$

In the expression on the left, the two operations are multiplication ($2 \bullet x$) and addition (+ 6). To solve the equation, the opposite operations are used to balance the equation.

Solving Linear Equations Concretely, Pictorially, and Symbolically

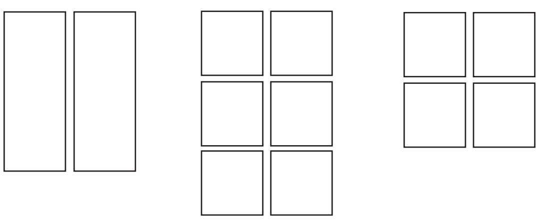
It is important for students to explore the solving of equations concretely, pictorially, and symbolically. An algebra balance (or pan balance) and algebra tiles are both appropriate concrete materials to use in demonstrating the preservation of equality.

Note: As algebra tiles have different colours, it is important to define which tile represents positive and which represents negative each time you use a new manipulative, and stay consistent with its meaning throughout its use.



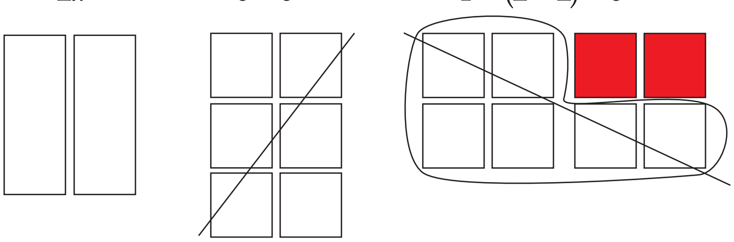
Solving Equations Concretely/Pictorially

$2x \quad + \quad 6 \quad = \quad 4$

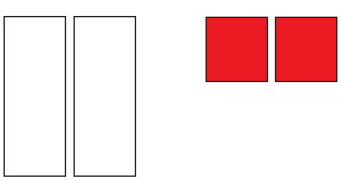


Remove (+6) from each side. There are not six (+1s) to remove from the right-hand side, so two zero pairs need to be added.

$2x \quad + \quad 6 - 6 \quad = \quad 4 + (2 - 2) - 6$

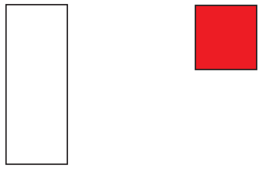


$2x \quad = \quad -2$



Simplify.

$x = -1$



Determine the value of each x.

Solving Equations Symbolically

$2x + 6 = 4$

$2x + 6 - 6 = 4 - 6$ Step 1: Subtract 6 from both expressions.

$2x = -2$

$\frac{2x}{2} = \frac{-2}{2}$ Step 2: Divide both expressions by 2.

$x = -1$

It is important to work with students to develop a procedure to solve equations so that they understand the mathematics involved. The important principles include the following:

- **Making zeros:** The reason you subtract 6 (in step 1 above) is to try to isolate the variable. 6 – 6 creates an addition of zero, leaving one term on the left-hand side. This is formally referred to as the *zero property of addition* or the *identity property of addition*.
- **Making ones:** The reason you divide by 2 (in step 2 above) is to try to isolate the variable, x. Dividing by 2 creates a multiplication by one, leaving just the variable on the left-hand side. This is formally referred to as the *inverse relationship* or the *identity property of multiplication*.
- **Preservation of equality:** To maintain balance and equality, you must do the same thing to both sides.

Distributive property is a property of real numbers that states that the product of the sum or the difference of two numbers is the same as the sum or difference of their products. Students should be familiar with the concept of distributive property as a multiplication strategy from Grade 5 [$a\ (b + c) = ab + ac$].

Examples:

Multiplication over Addition

2(15 + 4)
= 2 • 15 + 2 • 4
= 30 + 8

$2(2y + 3)$
$= 2 \bullet 2y + 2 \bullet 3$
$= 4y + 6$

Multiplication over Subtraction

$4(12 - 8)$
$= 4 \bullet 12 - 4 \bullet 8$
$= 48 - 32$

$3(5t - 3)$
$= 3 \bullet 5t - 3 \bullet 3$
$= 15t - 9$

Using the Distributive Property to Solve Equations

There are two different methods to solve the equation below. Neither is more correct than the other. Students should be encouraged to discover both, and use either freely on a case-by-case basis.

$2(x + 3) = 5$

$2 \bullet x + 2 \bullet 3 = 5$ Apply the distributive property.

$2x + 6 = 5$

$2x + 6 - 6 = 5 - 6$ Subtract 6 from both expressions.

$2x = -1$

$\frac{2x}{2} = \frac{-1}{2}$ Divide both expressions by 2.

$x = -\frac{1}{2}$

OR

$2(x + 3) = 5$

$\frac{2(x+3)}{2} = \frac{5}{2}$ Divide both expressions by 2.

$x + 3 = 2\frac{1}{2}$

$x + 3 - 3 = 2\frac{1}{2} - 3$ Subtract 3 from both sides.

$x = -\frac{1}{2}$

Note: Preservation of equality is the most important concept to consider when solving linear equations. It can be compared to a rule in a game. **Making zeros** and **making ones** are strategies for solving linear equations. When first solving linear equations, allow students to try operations, and then ask whether the operations or strategies were useful.

For example: solve $3b - 5 = 7$

A student may want to subtract 3 from each side.

Done correctly, this gives 3b – 8 = 4.

While this strategy is not wrong, it is not useful because there are still three terms in the equation and the student is no closer to isolating the variable, which is the "goal" of the "game."

Allowing students to use inefficient strategies at first and discussing how efficient they are will help students understand more efficient strategies, while not introducing too many rules to memorize.

Mathematical Language

balance
constant
equation
equivalent
evaluate
expression
formula
one-step linear equation
opposite operation
substitution
two-step linear equation
variable

Learning Experiences

Assessing Prior Knowledge

Materials: BLM 8.PR.2.1: Algebra Pre-Assessment, algebra tiles or balance scale, chips

Organization: Individual

Procedure:

1. Tell students that they will be extending their understanding of solving equations; however, you first need to determine what they already know about solving equations.
2. Hand out copies of BLM 8.PR.2.1: Algebra Pre-Assessment, and let students know what manipulatives are available to them. Have students solve the questions individually, showing their work.

Observation Checklist

☑ Observe students' responses to determine whether they can do the following:

- ❒ Model and solve problems that can be represented by one-step linear equations of the form $x + a = b$ concretely, where a and b are whole numbers.
- ❒ Model and solve problems that can be represented by one-step linear equations of the form $x - a = b$ pictorially, where a and b are whole numbers.
- ❒ Model and solve problems that can be represented by one-step linear equations of the form $ax = b$ pictorially, where a and b are whole numbers.
- ❒ Model and solve problems that can be represented by one-step linear equations of the form $\frac{x}{a} = b$ symbolically, where a and b are whole numbers and $a \neq 0$.
- ❒ Communicate mathematically.
- ❒ Apply mental mathematics and reasoning strategies in order to compute mathematically.

Suggestions for Instruction

- **Model a problem with a linear equation, and solve the equation using concrete models.**

Materials: Chart paper,
Kroll, Virginia. *Equal Shmequal: A Math Adventure*. Illus. Philomena O'Neill. Watertown, MA: Charlesbridge Publishing, Inc., 2005. Print.

Organization: Pairs/whole class

Procedure:

1. Tell students that you will read them a story called *Equal Shmequal*, in which the animals use a see-saw to make equal teams. Students will need to explain how the see-saw could be used to represent linear equations.
2. Read the story.
3. Have students pair up and use the see-saw to represent an equation of their own on chart paper.

4. Have students present their work to the class, and allow time for discussion. Where necessary, facilitate discussion using guiding questions such as the following:
   - When is a see-saw balanced?
   - When are numbers balanced?
   - If you make one side of the see-saw heavier, what happens?
   - If you make the value of one side of the equation higher, what happens?
   - How can you re-establish balance on a see-saw?
   - How can you re-establish balance in an equation?
5. Post students' work on the classroom wall so that students can refer to them.

Observation Checklist

☑ Observe students' responses to determine whether they can do the following:

- ❒ Communicate mathematically.
- ❒ Visualize a balance with symbolic algebraic representations.

Suggestions for Instruction

- **Model a problem with a linear equation, and solve the equation using concrete models.**

Materials: BLM 5–8.20: Algebra Tiles, paper ($8\frac{1}{2} \times 11$)

Organization: Whole class/individual

Procedure:

1. Remind students that in earlier grades they learned how to solve equations using positive whole numbers. Now they are going to learn how to solve equations using positive and negative integers.
2. Distribute $+x$, $-x$, +1, and –1 algebra tiles to all students. Review the representations of the algebra tiles. It does not matter which representation you choose to use, providing you are consistent.
3. Have students make an equal sign (=) on paper.
4. Write the following on the whiteboard: $4x = -16$.
5. Have students model the equation at their desks.
6. Ask students to determine the value of x by manipulating the algebra tiles. Discuss the various strategies students use to determine the value of x.

7. Always verify that the solution is correct. Start with $4x = -16$. Substitute -4 for x and determine that $4 \bullet -4 = -16$. Note that -16 will appear on both sides of the equal sign, so the solution is correct.
8. Repeat the process with simple equations until students feel comfortable using algebra tiles.
9. Repeat the process with equations containing negative x values, such as $-x + 3 = 2$, $\frac{-x}{2} = -2$.

   Note: These equations are more difficult because students will end up with a negative x equals a negative number. However, we are looking for the positive value of x. Students will need to apply their understanding of integers to this concept to determine the variable.
10. Write the following equations on the whiteboard and have students solve the equations concretely:

    $-2x = -8$, $x - 4 = -8$, $\frac{2x}{4} = 4$, $5x + 2 = 12$

Observation Checklist

☑ As students are working on the equations, circulate to observe whether they are able to do the following:

- ❒ Model and solve linear equations using models.

Suggestions for Instruction

- **Draw a visual representation of the steps used to solve a linear equation and record each step symbolically.**
- **Verify the solution to a linear equation using a variety of methods, including concrete materials, diagrams, and substitution.**

Materials: BLM 5–8.20: Algebra Tiles, white paper ($8\frac{1}{2} \times 11$)

Organization: Individual

Procedure:

1. Tell students that they will be building on what they learned in the previous learning experience. Ask them to model the following equation using algebra tiles: $2x + 4 = 10$
2. Provide each student with a white piece of paper and ask students to fold the paper in half.

3. Have them draw the equation on the top left-hand side of the paper, using the appropriate colours.
4. Ask them to continue with their diagram to solve the problem, working down the left side of the page.
5. When they have solved the problem pictorially, ask them to write the symbolic equation on the top right-hand side of the paper.
6. Have them work down the page, solving the equation so that each step of the symbolic solution matches the pictorial step across from it.
7. Always have students check their work when they have a solution to see whether the result balances the equation. Students will substitute the result for the variable and work through the equation to see whether it balances.

Observation Checklist

☑ As students are working on the equations, circulate to observe whether they are able to do the following:

- ❒ Model and solve linear equations pictorially.
- ❒ Model and solve linear equations symbolically.
- ❒ Verify the solutions of linear equations.

Suggestions for Instruction

- **Draw a visual representation of the steps used to solve a linear equation and record each step symbolically.**
- **Verify the solution to a linear equation using a variety of methods, including concrete materials, diagrams, and substitution.**

Materials: BLM 5–8.20: Algebra Tiles, brown lunch bags (2 per group), self-adhesive notes with a large equal sign (=) written on them

Organization: Pairs

Procedure:

1. Have students form groups of two. Give each pair an equal sign and two bags.
   - Bag A contains +1, –1, +x, and –x algebra tiles.
   - Bag B contains +1 and –1 algebra tiles.
2. Have partner A take a handful of tiles from bag A and place the tiles on one side of the equal sign.

3. Have partner B take a handful of tiles from bag B and place the tiles on the other side of the equal sign.
4. Have students work together to simplify and solve the equations concretely and record their process either pictorially or symbolically.

Observation Checklist

☑ Observe students' responses to determine whether they can do the following:

- ❒ Model and solve linear equations concretely.
- ❒ Model and solve linear equations pictorially.
- ❒ Model and solve linear equations symbolically.

Suggestions for Instruction

- **Solve a linear equation symbolically.**
- **Solve a linear equation by applying the distributive property (e.g., 2(*x* + 3) = 5; 2*x* + 6 = 5; . . .).**

Materials: BLM 8.PR.2.2: Solving Equations Symbolically

Organization: Individual/pairs/whole class

Procedure:

1. As a review of the distributive property, write the following on the whiteboard: 7 • 16. Ask students whether they can think of a simple way to determine this product. Record and discuss all ideas.
2. Record the following on the whiteboard: 2(*a* + 4). Is there another way to write this expression? Record and discuss students' responses.
3. Ask students whether they would be able to solve the equation if you made the expression 2(*a* + 4) into an equation, 2(*a* + 4) = 12. Record and discuss students' responses.
4. Provide each student with a copy of BLM 8.PR.2.2: Solving Equations Symbolically.
5. Have students use the Think-Pair-Share strategy, following these steps:
   - Complete the BLM individually.
   - Share responses with a partner.
   - Make notes of any questions that arise.
   - Find a new partner to share responses with.
6. Discuss students' results as a class and allow opportunity for questions as needed.

7. Ask students to solve, and verify their solutions to, the following questions in their math journals:

   $\frac{-x}{5} = -30$, $-4k = 24$, $3(y + 6) = 27$, $-4(r - 6) = 12$

Observation Checklist

☑ Observe students' responses to determine whether they can do the following:

- ❒ Solve linear equations symbolically.
- ❒ Solve linear equations by applying the distributive property.
- ❒ Verify the solutions of linear equations.

Suggestions for Instruction

- **Solve a linear equation symbolically.**
- **Solve a linear equation by applying the distributive property (e.g., $2(x + 3) = 5$; $2x + 6 = 5$; . . .).**

Materials: BLM 8.PR.2.3: Algebra Match-up, scrap paper

Organization: Pairs

Procedure:

1. Have students choose a partner. Provide each pair of students with a set of algebra match-up cards (copied from BLM 8.PR.2.3: Algebra Match-up).
2. Tell students that they will be playing *Algebra Match-up*. (**Note:** The game is played like *Go Fish*.)
3. Explain the rules for *Algebra Match-up*.
   - Deal 5 cards to each player.
   - Player A asks player B for the pair to one of his or her cards. For example, Player A asks for a card by stating, "Twice a number is equal to 6."
   - If player B has the solution, he or she will respond, "Yes, you may have $y = 3$," and will give player A that card. If player B does not have the solution, he or she will say, "No, take a card."
   - Play continues until all cards have been matched up.
   - The player with the most pairs at the end of the round wins.
   - Players are allowed to use scrap paper to try to determine matching pairs.
4. Have students play the game.

Observation Checklist

☑ Observe students' responses to determine whether they can do the following:

- ❒ Solve linear equations symbolically.
- ❒ Apply the distributive property in order to solve equations.
- ❒ Use mental mathematics and reasoning strategies to determine the solution to equations.

Suggestions for Instruction

- **Identify and correct errors in an incorrect solution of a linear equation.**

Materials: BLM 8.PR.2.4: Analyzing Equations, chart paper, BLM 8.PR.2.5: Analyzing Equations Assessment

Organization: Pairs/whole class/individual

Procedure:

1. Hand out copies of BLM 8.PR.2.4: Analyzing Equations.
2. Have students, working in pairs, analyze the solutions to the equations presented on the BLM. Ask them to determine whether or not the equations were solved correctly. For any equations that were not solved correctly, students must record the error and provide a corrected solution on chart paper. Students need to be prepared to share their responses with the class.
3. As a class, discuss the solutions presented by students. Post the correct solutions on the classroom wall.
4. Have students create their own linear equation and provide a solution, either with or without an error. Have each student exchange his or her example with a partner, who must review the solution to determine whether or not there is an error. Partners share their findings with each other.
5. Provide students with BLM 8.PR.2.5: Analyzing Equations Assessment. Ask them to analyze the solutions, determine whether they are correct or incorrect, and explain their reasoning.

Observation Checklist

☑ Observe students' responses to determine whether they can do the following:

- ❐ Identify and correct errors in an incorrect solution of a linear equation.
- ❐ Solve equations symbolically.
- ❐ Communicate solution errors using mathematical language.

Suggestions for Instruction

- **Solve a problem using a linear equation, and record the process.**

Materials: BLM 8.PR.2.6: Solving Problems Using a Linear Equation, chart paper

Organization: Small group/whole class

Procedure:

1. Tell students that they will be applying what they know about solving equations to problem-solving scenarios.
2. Divide the class into small groups, and provide students with copies of BLM 8.PR.2.6: Solving Problems Using a Linear Equation.
3. Have students work together to determine methods of solving the problems presented on the BLM. Have the groups present their solution methods to the class.
4. As a class, discuss the various solution methods presented. If a group does not come up with an algebraic method of solving the problems, offer that as a possible solution.
5. Tell students that it is important for them to practise representing word problems algebraically. Offer several more problems for students to represent algebraically (but not to solve them).
6. Have students, working in small groups, develop scenarios in which one would use linear equations to solve the problems, and have them record their problems on chart paper.
7. Have groups exchange problems and use algebraic reasoning to solve the problems.
8. Present, discuss, and post the problems and solutions generated by students.

Observation Checklist

☑ Observe students' responses to determine whether they can do the following:

- ❒ Solve problems using a linear equation.
- ❒ Demonstrate an understanding of preservation of equality by modelling preservation of equality.

PUTTING THE PIECES TOGETHER

Business and Marketing Analyst

Introduction:

This task allows students to put themselves in the role of a business and marketing analyst whose job is to determine how much of a product must be sold to break even, how much profit will be made if the product is sold out, and how much to sell the product for, making comparisons and setting a price for the product.

Purpose:

Students will be able to solve one- and two-step linear equations and use numbers to compare profit and loss.

Curricular Links: ELA

Materials/Resources: Calculator, white paper for business report, poster paper (if required for presentation)

Organization: Individual/small group

Scenario:

- You are a business and marketing analyst for Jamie Lee Foods.
- You know that it costs the company $250 to bake 100 dozen cookies.
- It will cost the company $250 to package the cookies individually.
- It will cost the company $175 to package the cookies in groups of 6.
- You need to determine the following:
  - If the cookies are sold individually for 50 cents each, how many cookies would be needed to cover all expenses (break even)? How much profit would be made if all the cookies were sold? Is selling the cookies for 50 cents each reasonable, or would it be better to sell them for 75 cents each?
  - If the cookies are sold as a 6-pack, what would be a reasonable sale price? Explain.

- You must prepare a written report that will be shared with the president and vice-president of Jamie Lee Foods. You must show all calculations and provide clear and concise explanations for determining what you think would be the best option for Jamie Lee Foods.

Assessment:

The following rubric can be used to assess achievement of the mathematics learning outcomes.

Criteria	Meeting Expectations	Developing to Meet Expectations	Beginning to Meet Expectations	Incomplete
The student				
■ sets up linear equations	❒ sets up all linear equations correctly using numbers and variables to calculate solutions	❒ sets up some linear equations correctly using numbers and variables to calculate solutions	❒ sets up few linear equations using numbers and variables to calculate solutions	❒ does not set up linear equations to show calculations
■ solves linear equations	❒ makes correct calculations that clearly communicate the mathematical processes required to determine the various options	❒ makes some errors in calculations and/or is somewhat disorganized when communicating the mathematical processes required to determine the various options	❒ makes many errors in calculations and/or is very disorganized when communicating the mathematical processes required to determine the various options	❒ does not provide calculations
■ analyzes costs to determine the best option for the company	❒ provides clear and organized communication of cost breakdowns and decisions for the best option	❒ provides somewhat unclear and disorganized communication of cost breakdowns and decisions for the best option	❒ provides very unclear and disorganized communication of cost breakdowns and decisions for the best option	❒ does not include a cost analysis

Extension:

Students could come up with an actual business plan on a smaller scale and then make and sell the product and donate profits to charity.

NOTES

Grade 8 Mathematics

Shape and Space

Shape and Space (Measurement and 3-D Objects and 2-D Shapes)—8.SS.2, 8.SS.3, 8.SS.4, 8.SS.5

Note: Specific learning outcome **8.SS.1** is addressed in the discussion of Number and Shape and Space (Measurement).

Enduring Understandings:

Many geometric properties and attributes of shapes are related to measurement.

The area of some shapes can be used to develop the formula for the area, surface area, and volume of other shapes.

While geometric figures are constructed and transformed, their proportional attributes are maintained.

All measurements are comparisons.

Length, area, volume, capacity, and mass are all measurable properties of objects.

The unit of measure must be of the same nature as the property being measured.

General Learning Outcomes:

Use direct or indirect measurement to solve problems.

Describe the characteristics of 3-D objects and 2-D shapes, and analyze the relationships among them.

Specific Learning Outcome(s):	Achievement Indicators:
8.SS.2 Draw and construct nets for 3-D objects. [C, CN, PS, V]	➛ Match a net to the 3-D object it represents. ➛ Construct a 3-D object from a net. ➛ Draw nets for a right circular cylinder, right rectangular prism, and right triangular prism, and verify [that the nets are correct] by constructing the 3-D objects from the nets. ➛ Predict 3-D objects that can be created from a net and verify the prediction.

continued

Specific Learning Outcome(s):	**Achievement Indicators:**
8.SS.3 Determine the surface area of ■ right rectangular prisms ■ right triangular prisms ■ right cylinders to solve problems. [C, CN, PS, R, V]	➛ Explain, using examples, the relationship between the area of 2-D shapes and the surface area of a 3-D object. ➛ Identify all the faces of a prism, including right rectangular and right triangular prisms. ➛ Describe and apply strategies for determining the surface area of a right rectangular or right triangular prism. ➛ Describe and apply strategies for determining the surface area of a right cylinder. ➛ Solve a problem involving surface area.
8.SS.4 Develop and apply formulas for determining the volume of right prisms and right cylinders. [C, CN, PS, R, V]	➛ Determine the volume of a right prism, given the area of the base. ➛ Generalize and apply a rule for determining the volume of right cylinders. ➛ Explain the relationship between the area of the base of a right 3-D object and the formula for the volume of the object. ➛ Demonstrate that the orientation of a 3-D object does not affect its volume. ➛ Apply a formula to solve a problem involving the volume of a right cylinder or a right prism.
8.SS.5 Draw and interpret top, front, and side views of 3-D objects composed of right rectangular prisms. [C, CN, R, T, V]	➛ Draw and label the top, front, and side views of a 3-D object on isometric dot paper. ➛ Compare different views of a 3-D object to the object. ➛ Predict the top, front, and side views that will result from a described rotation (limited to multiples of 90°) and verify predictions. ➛ Draw and label the top, front, and side views that result from a rotation (limited to multiples of 90°). ➛ Build a 3-D block object, given the top, front, and side views, with or without the use of technology. ➛ Sketch and label the top, front, and side views of a 3-D object in the environment, with or without the use of technology.

Prior Knowledge

Students may have had experience with the following:

- Demonstrating an understanding of measuring length (cm, m) by
  - selecting and justifying referents for the units cm and m
  - modelling and describing the relationship between the units cm and m
  - estimating length using referents
  - measuring and recording length, width, and height
- Describing 3-D objects according to the shape of the faces, and the number of edges and vertices
- Demonstrating an understanding of area of regular and irregular 2-D shapes by
  - recognizing that area is measured in square units
  - selecting and justifying referents for the units cm^2 or m^2
  - estimating area by using referents cm^2 or m^2
  - determining and recording area (cm^2 or m^2)
  - constructing different rectangles for a given area (cm^2 or m^2) in order to demonstrate that many different rectangles may have the same area
- Solving problems involving 2-D shapes and 3-D objects
- Describing and constructing rectangular and triangular prisms
- Demonstrating an understanding of volume by
  - selecting and justifying referents for cm^3 or m^3 units
  - estimating volume by using referents for cm^3 or m^3
  - measuring and recording volume (cm^3 or m^3)
  - constructing rectangular prisms for a given volume
- Describing and providing examples of edges and faces of 3-D objects, and sides of 2-D shapes, that are
  - parallel
  - intersecting
  - perpendicular
  - vertical
  - horizontal
- Developing and applying a formula for determining the
  - perimeter of polygons
  - area of rectangles
  - volume of right rectangular prisms

- Developing and applying a formula for determining the area of
  - triangles
  - parallelograms
  - circles

Related Knowledge

Students should be introduced to the following:

- Demonstrating an understanding of perfect squares and square roots, concretely, pictorially, and symbolically (limited to whole numbers)

Background Information

Measurement

The key to understanding measurement is developing an understanding of the formulas for calculating surface area and volume and then being able to use the formulas to solve problems.

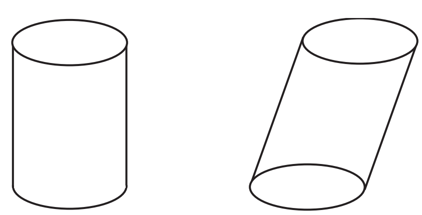
Determining the surface area and volume of right cylinders, right rectangular prisms, and right triangular prisms is an extension of already known formulas (area of a rectangle, area of a triangle, area of a circle, circumference of a circle, and volume of a rectangle) and the nets of these 3-D objects.

Definitions

cylinder

A geometric figure with two parallel and congruent, flat (plane) surfaces connected by one curved surface (curved face).

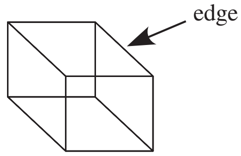
Examples:



edge

A line segment where two faces of a 3-D figure intersect.

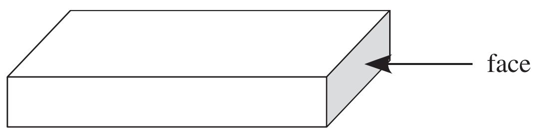
Example:



face

A flat surface of a solid.

Example:

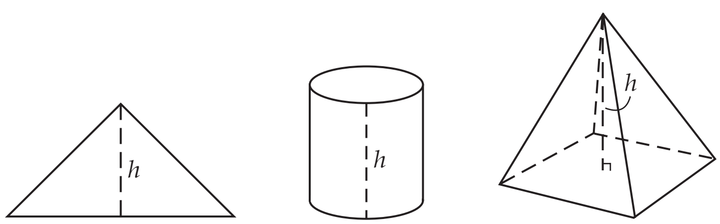


height

Can be used in the following ways:

- The measurement from base to top.
- The perpendicular distance from a vertex to the line containing the opposite side of a plane figure; the length of a perpendicular from the vertex to the plane containing the base of a pyramid or cone; the length of a perpendicular between the planes containing the bases of a prism or cylinder.

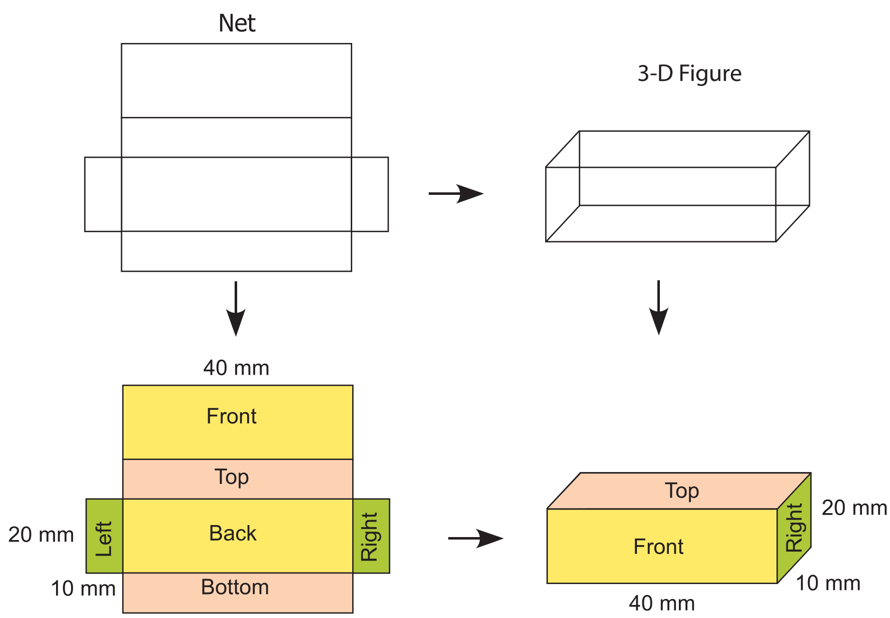
Examples:



net

The 2-D set of polygons of which a 3-D object is composed.

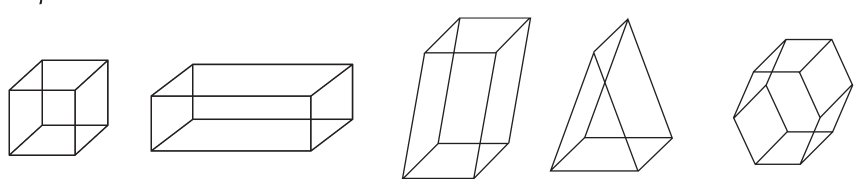
Example:



prism

A 3-D figure (solid) that has two congruent and parallel faces that are polygons (the bases); the remaining faces are parallelograms. The name of the prism is determined by the shape of the base.

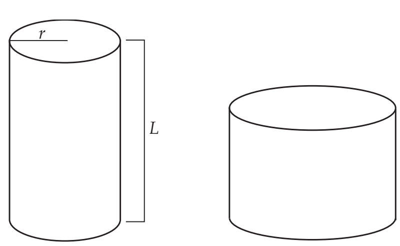
Examples:



right cylinder

A geometric figure with two parallel and congruent, flat (plane) surfaces connected at a right angle by one curved surface (curved face). A right cylinder has a 90° angle where the base and height meet.

Examples:



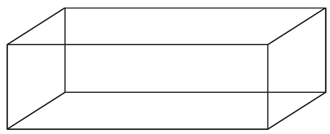
right prism

A prism that has a 90° angle where the base and height meet.

right rectangular prism

A prism whose six faces are rectangles; a prism with a rectangular base.

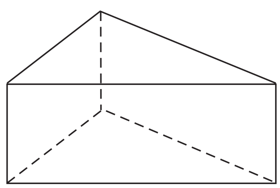
Example:



right triangular prism

A prism with a triangular base whose faces meet the base at right angles.

Example:



surface area

The sum of the areas of the faces or curved surface of a 3-D object.

three-dimensional (3-D) object

An object that has length, width, and height (e.g., prism, pyramid, cylinder, cone).

two-dimensional (2-D) shape

A figure that has two measures, such as length, width, or height (e.g., circle, square, triangle).

vertex

Can be used in the following ways:

- The common endpoint of two sides of a polygon.
- The common endpoint of two rays that form an angle.
- The common point where three or more edges of a 3-D solid meet.

view

A 2-D representation of a 3-D object.

volume

- In general, volume refers to an amount of space occupied by an object (e.g., solids, liquids, gas).
- In science, volume is expressed in cubic units (e.g., cubic centimetres (cm^3) and cubic metres (m^3)).
- In mathematics, volume means the same thing as *capacity*. Both volume and capacity are represented by the number of cubes (and parts of cubes) of a given size it takes to fill an object.

Determining Surface Area

Surface Area of a Right Prism

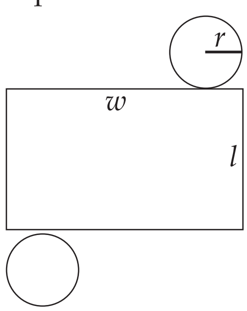
To determine the surface area of the right prism, determine the areas of each face and then add the areas together.

- **Surface Area of a Right Cylinder**

  To determine the surface area of a right cylinder, the shapes that make up the cylinder must be known. If you look at the net of a right cylinder, you will find that the shapes of the right cylinder are two circles (if there is a top and a bottom) and a rectangle.

  Example:

  A possible net of a right cylinder looks like this:



Area of a rectangle

$l \bullet w$

Area of a cirlce

πr^2

To determine the surface area of the right cylinder, determine the areas of the rectangle and the two circles, and then add the areas together.

Surface Area of Right Cylinder

= Area of rectangle + area of circles

$= (l \bullet w) + 2(\pi r^2)$

Surface area is measured in square units, written as cm^2, m^2, and so on.

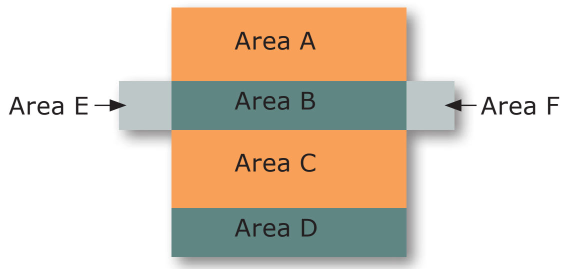
Note: The length of the rectangle is the same as the circumference of the circle.

- **Surface Area of a Right Rectangular Prism**

  To determine the surface area of a right rectangular prism, the shapes that make up the rectangular prism must be known. If you look at the net of a right rectangular prism, you will find that the shapes of the right rectangular prism are six rectangles, with opposite sides of the boxes the same.

  Example:

  A possible net of a right rectangular prism looks like this:



Area of a rectangle

$l \bullet w$

  To determine the surface area of the right rectangular prism (Area A = Area C, Area B = Area D, Area E = Area F), you need to determine the areas of all six rectangles. Since opposite sides are equal, you only have to calculate the area of three rectangles, double each area, and add them.

  Surface Area of Right Rectangular Prism

  = 2(Area A) + 2(Area B) + 2(Area E)

  $= 2(l_A + w_A) + 2(l_B + w_B) + 2(l_E + w_E)$

  Note: l_A is the length of rectangle A, while l_B is the length of rectangle B. The length of rectangle A may or may not be the same as the length of rectangle B. Students need to be careful to use the correct dimensions to find each area. They are not expected to use the notation l_A.

  Surface area is measured in square units, written as cm^2, m^2, and so on.

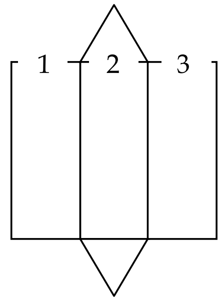
- **Surface Area of Right Triangular Prism**

  To determine the surface area of a right triangular prism, the shapes that make up the triangular prism must be known. If you look at the net of a right triangular prism, you will find that the shapes of the right triangular prism are three rectangles and two triangles, with the opposite triangles being the same size.

Example:

A possible net of a right triangular prism looks like this:

The rectangles may or may not be the same size, depending on the type of triangle the base is made from.

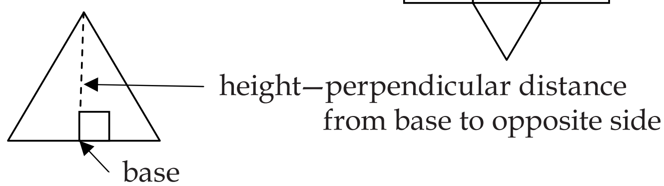


Area of a rectangle

$l \bullet w$

Area of a triangle

$\dfrac{b \bullet h}{2}$



To determine the surface area of the right triangular prism, you need to determine the area of the two triangles and the area of the three rectangles. You may be able to combine some areas if they contain similar measurements. A general formula for determining the surface area of a right triangular prism is as follows:

Surface Area of a Right Triangular Prism

= (area of rectangle 1) + (area of rectangle 2) + (area of rectangle 3) + 2(area of triangle)

$= (l_1 \bullet w_1) + (l_2 \bullet w_2) + (l_3 \bullet w_3) + 2(\dfrac{b \bullet h}{2})$

Note: $l_1 \Rightarrow$ length of rectangle 1; $w_1 \Rightarrow$ width of rectangle 1.

Determining Volume

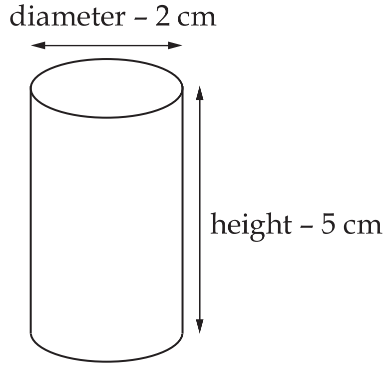
Volume of a Right Prism

To determine the volume of a right prism, determine the area of the base and multiply it by the height.

- **Volume of a Right Cylinder**

  The volume of a right cylinder is determined by multiplying the area of the base by the height of the cylinder.

Example:



Area of base $= \pi r^2$

$= \pi\ (1 \text{ cm})^2$

$= 3.14 \text{ cm}^2$

Volume = Area of base • height

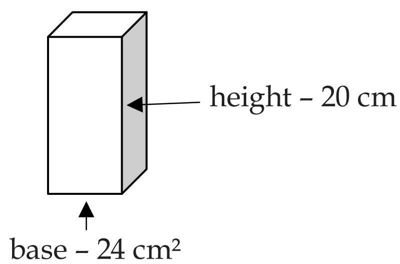
$= 3.14 \text{ cm}^2 \bullet 5 \text{ cm}$

$= 15.7 \text{ cm}^3$

- **Volume of a Right Rectangular Prism**

  Volume of a right rectangular prism is determined by multiplying the area of the base of the rectangular prism by the height of the rectangular prism.

  Example:

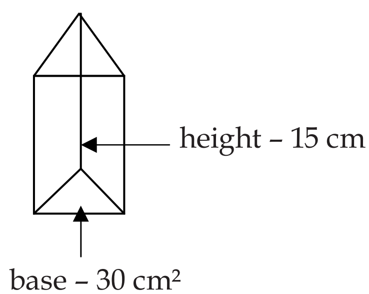


Volume of a rectangular prism
= area of base times height
$= 24\text{ cm}^2 \bullet 20\text{ cm}$
$= 480\text{ cm}^3$

- **Volume of a Right Triangular Prism**

  The volume of a right triangular prism is determined by multiplying the area of the base of the triangular prism by the height of the triangular prism.

  Example:



Volume of a triangular prism
= area of base times height
$= 30\text{ cm}^2 \bullet 15\text{ cm}$
$= 450\text{ cm}^3$

Mathematical Language

2-D shapes	net
3-D objects	orientation of a shape
area	radius
base of a prism	right cylinder
diameter	right rectangular prism
edge	right triangular prism
face	vertex
formula	view
height of a prism	volume

Assessing Prior Knowledge

Materials: BLM 8.SS.2.1: Measurement Pre-Assessment

Organization: Individual

Procedure:

1. Tell students that they will be extending their understanding of measurement over the next few lessons; however, you first need to find out what they already know about measurement.
2. Hand out copies of BLM 8.SS.2.1: Measurement Pre-Assessment and have students complete it individually, showing their work.

Observation Checklist

☑ Observe students' responses to determine whether they can do the following:
- ❒ Determine the area of a rectangle.
- ❒ Determine the circumference of a circle.
- ❒ Determine the volume of a right rectangular prism.
- ❒ Determine the area of a circle.
- ❒ Determine the area of a triangle.

Suggestions for Instruction

- **Draw and label the top, front, and side views of a 3-D object on isometric dot paper.**
- **Sketch and label the top, front, and side views of a 3-D object in the environment, with or without the use of technology.**

Materials: Cereal or macaroni boxes (ideally, multiples of the same type and size of boxes), cubes/blocks, BLM 5–8.21: Isometric Dot Paper

Organization: Whole class/pairs/individual

Procedure:

1. Tell students that they will be learning about nets for 3-D objects.
2. Discuss the meaning of the following terms with the class:
   - 2-dimensional
   - 3-dimensional
   - net
   - net of a 3-D object
   - view (with respect to 3-D objects)
3. Place boxes around the room, showing different faces of the boxes. Have students draw the boxes from where they are sitting, so they will be drawing different views.
4. Have students, working in pairs, each choose one of their drawings. Without showing their drawings to each other, students take turns explaining how to draw their respective views while the partners try to replicate the drawings based on the explanations. Ask those students who were successful in having their partners draw an exact replica of their drawings, what key words they used to help their partners. (Observe whether students are able to use the terms *face*, *edge*, and *vertex* in their descriptions.)
5. Show students a right rectangular prism. To describe 3-D objects, one needs to count the number of faces, edges, and vertices on the objects.
   - A *face* is a flat or curved surface.
   - An *edge* is a line segment where two faces meet.
   - A *vertex* is a point where three or more edges meet.
6. Provide each student with a copy of BLM 5–8.21: Isometric Dot Paper. Ask students to redraw their boxes on the dot paper and identify the faces, edges, and vertices of their boxes.
7. Ask students the following questions:
   - What is the front of your box? What is the side? What is the top? Does it matter?
   - Were you able to see all the views when you labelled your box? Do you need to see all the views?
   - Why do you need to have only one side view if the top and front views are given?
8. Provide each student with 5 to 10 cubes/blocks. Have each student create an object and then draw and label the front, side, and top views of the object on the isometric dot paper provided (see BLM 5–8.21: Isometric Dot Paper).

Note: Students may need some time to explore how to use isometric dot paper. The dots are at angles, so students will need some time to learn how to connect the dots to draw and label their objects.

Observation Checklist

☑ Observe students' responses to determine whether they can do the following:

- ❒ Draw and label the top, front, and side views of a 3-D object on isometric dot paper.
- ❒ Visualize 3-D objects in order to represent them in a 2-D picture.

Suggestions for Instruction

- **Compare different views of a 3-D object to the object.**
- **Predict the top, front, and side views that will result from a described rotation (limited to multiples of 90°) and verify predictions.**
- **Draw and label the top, front, and side views that result from a rotation (limited to multiples of 90°).**
- **Sketch and label the top, front, and side views of a 3-D object in the environment, with or without the use of technology.**

Materials: Various 3-D objects (e.g., books, rectangular erasers, boxes, CD cases) that students have gathered, BLM 5–8.21: Isometric Dot Paper, cubes, overhead or LCD projector

Organization: Individual/whole class

Procedure:

1. Ask each student to select one object from the assortment that has been gathered. Then have students do the following:
   - Draw and label the top, front, and side views of the object on the isometric dot paper provided.
   - Rotate the object 90° clockwise and draw and label the top, front, and side views of the object on the isometric dot paper.
   - Compare the different views of the 3-D object.
2. Show students a picture of any type of box using an overhead or LCD projector. Have them draw and label the top, front, and side views that result from a 270° clockwise turn. (Students may note that a 270° clockwise turn is the same as a 90° counter-clockwise turn.)
3. Show students a 2-D view of one of the objects and have them determine which object it could be.

Observation Checklist

☑ Observe students' responses to determine whether they can do the following:

- ❒ Draw and label top, front, and side views of a 3-D object that has been rotated.
- ❒ Predict the top, front, and side views of an object to be rotated.
- ❒ Visualize the new image.
- ❒ Analyze the original view to determine the new view.
- ❒ Draw and label the views as they rotate.

Suggestions for Instruction

- **Draw and label the top, front, and side views of a 3-D object on isometric dot paper.**
- **Build a 3-D block object given the top, front, and side views, with or without the use of technology.**

Materials: Interlocking blocks/cubes, BLM 5–8.21: Isometric Dot Paper

Organization: Pairs

Procedure:

1. Pair up students in the class.
2. Have each student take approximately 20 blocks/cubes.
3. Ask each student to do the following:
   - Use six cubes to make a 3-D object and keep it hidden from your partner.
   - Use isometric dot paper to draw and label the top, front, and side views of your 3-D shape.
   - Exchange your isometric dot paper views with those of your partner and have your partner build the 3-D object you created.
   - Compare the original 3-D object with the partner's representation of it.
4. For the next rounds, increase the number of blocks to eight, and then to ten.

Observation Checklist

☑ Observe students' responses to determine whether they can do the following:

- ❐ Draw and label the top, front, and side views of a 3-D object.
- ❐ Build a 3-D block from the top, side, and front views.

Suggestions for Instruction

- **Construct a 3-D object from a net.**
- **Draw nets for a right circular cylinder, right rectangular prism, and right triangular prism, and verify [that the nets are correct] by constructing the 3-D objects from the nets.**
- **Predict 3-D objects that can be created from a net, and verify the prediction.**
- **Match a net to the 3-D object it represents.**

Materials: BLM 8.SS.2.2: Nets of 3-D Objects, white paper

Organization: Individual

Procedure:

1. Hand out copies of BLM 8.SS.2.2: Nets of 3-D Objects. Have students, working individually, predict what a net will make, and then have them construct the object to verify their prediction.
2. Repeat the first step with a number of nets so that students are able to see how the nets make the 3-D objects.
3. Place the following three names in a container: right circular cylinder, right rectangular prism, and right triangular prism. Ask students to pick one of the names out of the container and draw the net for the selected 3-D object, using the white paper provided.
4. Have students construct a 3-D object from their net to determine whether their net is correct.
5. Repeat the process to observe whether students can construct nets for all three 3-D objects.

Observation Checklist

☑ Observe students' responses to determine whether they can do the following:

- ❒ Predict a 3-D object based on the net.
- ❒ Construct a net for a right circular cylinder.
- ❒ Construct a net for a right rectangular prism.
- ❒ Construct a net for a right triangular prism.

Suggestions for Instruction

- **Construct a 3-D object from a net.**
- **Match a net to the 3-D object it represents.**

Materials: BLM 8.SS.2.3: 3-D Objects, white paper ($8\frac{1}{2} \times 11$), BLM 8.SS.2.4: Matching

Organization: Whole class/small group/individual

Procedure:

1. Ask students to suggest examples of 3-D objects. Record their suggestions on the whiteboard.
2. Hand out copies of BLM 8.SS.2.3: 3-D Objects. Ask students whether they notice anything about the objects represented that would help them determine the names of the objects.
3. Have students, working in small groups, draw nets of the 3-D objects shown on BLM 8.SS.2.3: 3-D Objects, using the white paper provided.
4. Ask students to construct the 3-D objects from the nets they drew to see whether they work.
5. Have students share their experience of creating the nets and constructing the 3-D object.
6. Hand out copies of BLM 8.SS.2.4: Matching. Have students complete the sheet individually.

Observation Checklist

☑ Observe students' responses to determine whether they can do the following:

- ❒ Construct a net from a 3-D object.
- ❒ Match a net to the 3-D object it represents.

Suggestions for Instruction

- **Match a net to the 3-D object it represents.**

Materials: Various 3-D objects (cube, rectangular prism, triangular prism, trapezoid prism, cylinder, square pyramid, triangular pyramid), white paper ($8\frac{1}{2} \times 11$)

Organization: Individual/whole class

Procedure:

1. Have each student select from a container one of the following objects: cube, rectangular prism, triangular prism, trapezoid prism, cylinder, square pyramid, or triangular pyramid.
2. Have students, working individually, draw a net for their selected objects.
3. Using students' nets, construct a scavenger hunt around the class. Students need to locate and identify each net that was created.

Observation Checklist

☑ Observe students' responses to determine whether they can do the following:

- ❒ Identify 3-D objects from their nets.

Suggestions for Instruction

- **Explain, using examples, the relationship between the area of 2-D shapes and the surface area of a 3-D object.**

Materials: Math journals

Organization: Whole class/individual

Procedure:

1. Tell students that they have constructed 3-D objects from nets, and now they are going to see how the area of 2-D shapes is connected to the surface area of 3-D objects.

2. Ask students: What is *area*? Have a conversation with students if they say area is length times width. Length times width is the way to calculate area (for a limited few 2-D shapes), but *area* is actually the number of square units that cover the surface that lies within a 2-D shape.
3. Come up with a class definition of *surface area*.
4. Have students respond to the following question in their math journals:

   Explain, using words and diagrams, the relationship between area and surface area. Give examples.

Observation Checklist

☑ Observe students' math journal responses to determine whether they can do the following:

- ❒ Relate the area of 2-D shapes with the surface area of 3-D objects.

Suggestions for Instruction

- **Explain, using examples, the relationship between the area of 2-D shapes and the surface area of a 3-D object.**
- **Identify all the faces of a prism, including right rectangular and right triangular prisms.**
- **Describe and apply strategies for determining the surface area of a right rectangular or right triangular prism.**

Materials: BLM 8.SS.3.1: Nets, square tiles, BLM 5–8.9: Centimetre Grid Paper (copied onto transparency), rulers

Organization: Pairs/whole class/individual

Procedure:

1. Pair up students, and provide each pair with two copies of a net of a right rectangular prism.
2. Have pairs use one net to construct the 3-D object.
3. Ask students to come up with a procedure for determining the surface area of the object. Let them know that square tiles, centimetre grid paper, and rulers are available if they need them.
4. As a class, discuss the various procedures that students used.
5. Repeat the process with right triangular prisms.
6. Have individual students find an example of a right triangular prism or a right rectangular prism in the classroom and find its surface area.

Observation Checklist

☑ Observe students' responses to determine whether they can do the following:

- ❒ Communicate mathematically.
- ❒ Apply reasoning skills to develop a procedure for determining the surface area of a 3-D shape.
- ❒ Calculate the surface area of prisms.

Suggestions for Instruction

- **Describe and apply strategies for determining the surface area of a right cylinder.**

Materials: White paper, a variety of cylinders, rulers, string, a camera, word processing software

Organization: Whole class/pairs

Procedure:

1. Briefly review how to determine the area of a circle.
2. Pair up students and tell them that they will be working together to determine the surface area of a cylinder. Have each pair select one cylinder to work with.
3. Students can use paper, rulers, string, or anything else in the class they would like to use to help them determine the surface area of their selected cylinder.
4. Have students take pictures to document their steps, import these pictures into a word processor, and describe the process they followed.

Observation Checklist

☑ Observe students' responses to determine whether they can do the following:

- ❒ Communicate mathematically.
- ❒ Describe a strategy for determining the surface area of a cylinder.

- **Describe and apply strategies for determining the surface area of a right cylinder.**

Materials: White paper

Organization: Individual

Procedure:

1. Provide each student with a piece of white paper.
2. Hold a sheet of paper in the landscape orientation, roll it to form a cylinder shape, and tape the paper together.
3. Ask students the following questions:
   - What was the original shape before the cylinder was formed? (rectangle)
   - How do you calculate the area of a rectangle? (length times width)
   - What is the length of the rectangle? (the height of the cylinder)
   - What is the width of the rectangle? (the circumference of the circle)
   - How do you calculate the circumference of a circle? ($\pi \bullet d$)
   - How do you calculate the area of the rectangular portion of the cylinder? ($\pi \bullet d \bullet h$)
   - What are the shapes at both ends of the cylinder? (circles)
   - How do you calculate the area of a circle? ($\pi \bullet r^2$)
   - How many circles are on a cylinder? (two)
   - How do you calculate the total surface area of a cylinder? [$2 (\pi \bullet r^2) + (h (\pi \bullet d))$]

Note: Students use the strategy for determining the surface area of a cylinder, but they do not need to memorize the formula.

Observation Checklist

☑ Observe students' responses to determine whether they can do the following:

- ❒ Describe a method for determining the surface area of a cylinder.
- ❒ Apply a strategy for determining the surface area of a cylinder.
- ❒ Calculate the surface area of the cylinder.

Suggestions for Instruction

- **Solve a problem involving surface area.**

Materials: BLM 8.SS.3.2: Surface Area Problems, chart paper, math journals

Organization: Small group/whole class/individual

Procedure:

1. Have students form small groups, and provide them with copies of BLM 8.SS.3.2: Surface Area Problems.
2. Assign each group one of the surface area problems. Ask the groups to solve their assigned problem and record their answer on the chart paper provided. They must explain what method they chose for solving the problem, why they chose that method, and why they think their answer is reasonable.
3. Have each group present their problem and solution to the class. Provide opportunities for the other groups to ask questions and add to the responses.
4. Ask students to create and solve a new surface area problem in their math journals.

Observation Checklist

☑ Observe students' responses to determine whether they can do the following:

- ❒ Solve surface area problems individually.
- ❒ Apply strategies for determining the surface area of 3-D objects.
- ❒ Communicate mathematically.

Suggestions for Instruction

- **Explain the relationship between the area of the base of a right 3-D object and the formula for the volume of the object.**
- **Determine the volume of a right prism, given the area of the base.**

Materials: A variety of boxes (e.g., cereal, facial tissue), centimetre cubes, rulers

Organization: Pairs/whole class

Procedure:

1. Pair up students, and have each pair select a box.
2. Let students know that their goal is to figure out a strategy to determine the volume of the box they have selected.
3. Have students share their strategies with the whole class. Ask guiding questions, such as the following:
   - How many centimetre cubes fill the bottom of your box? What is the area of the bottom of your box?
   - How many centimetre cubes stack up the corner of your box? What is the height of your box?
   - Can you use that information to determine the volume of your box? Explain.
   - Can you just use measurements to determine the volume of the box? Explain.
   - Why is it important to know the area of the base of the box in order to determine its volume?

Observation Checklist

☑ Observe students' responses to determine whether they can do the following:

- ❒ Develop a strategy to determine the volume of a box.

Suggestions for Instruction

- **Explain the relationship between the area of the base of a right 3-D object and the formula for the volume of the object.**
- **Determine the volume of a right prism, given the area of the base.**

Materials: Rulers

Organization: Whole class/individual

Procedure:

1. Tell students that they will now be exploring how to calculate the volume of right rectangular prisms and right triangular prisms.
2. Ask students the following questions:
   - What is volume? (*Volume* is the amount of space an object occupies. It is measured in cubic units.)

- Have you ever had to determine the volume of an object before? (Some students may say they have determined the volume of a cube by multiplying length • width • height.)
- Can you draw a cube and label the length, width, and height of the cube? Have students do this individually.
- If we multiply length by width, what do we determine? (the area of the base)
- Where have we calculated areas before? (when finding surface area)
- So, if we think of the prisms we have been working with, how can we determine the volume of the prisms? (area of the base times height of the prism)

3. Draw different rectangular and triangular prisms on the whiteboard. Write the area of the base and the height of each prism. Have students calculate the volume.

Observation Checklist

☑ Observe students' responses to determine whether they can do the following:

- ❒ Connect their prior knowledge of area and surface area in order to determine the volume of right rectangular and right triangular prisms.
- ❒ Use the given information to calculate the volume of the prisms.

Suggestions for Instruction

- **Generalize and apply a rule for determining the volume of right cylinders.**

Materials: Paper

Organization: Whole class/pairs/individual

Procedure:

1. Draw at least four different cylinders on the whiteboard and label the area of the base, the height, and the volume.
2. Have students make a table and label the cylinder number, the radius, the area of the base, the height, and the volume.
3. Have students work with partners to see whether they can determine the relationship between the numbers provided. (The relationship is the area of the base times the height equals the volume.)

4. Provide students with four more cylinders with the area of the base and the height provided. Have them, independently, determine the volume of the right cylinder.

Observation Checklist

☑ Observe students’ responses to determine whether they can do the following:

- ❒ Generalize a rule for determining the volume of a cylinder.
- ❒ Connect the area of a circle with the volume of a cylinder.

Suggestions for Instruction

- **Demonstrate that the orientation of a 3-D object does not affect its volume.**

Materials: Variety of 3-D objects (2 identical objects of each shape) taped to tabletops in differing orientations, math journals

Organization: Pairs/whole class/individual

Procedure:

1. Have students, working in pairs, determine the volume of a variety of specified 3-D objects and make a note of anything interesting they discover as they determine the volumes.
2. Ask students to record their measurements, calculations, and observations in an organized fashion.
3. Discuss students’ interesting discoveries as a class.
4. Have students explain, in their math journals, why orientation of a 3-D object does not affect its volume.

Observation Checklist

☑ Observe students’ responses to determine whether they can do the following:

- ❒ Determine the volume of 3-D objects.
- ❒ Explain why orientation does not affect volume.

- **Apply a formula to solve a problem involving the volume of a right cylinder or a right prism.**

Materials: BLM 8.SS.4.1: Volume Problems, chart paper, math journals

Organization: Small group/whole class/individual

Procedure:

1. Tell students that they will be solving problems that involve right cylinders and right prisms.
2. Divide students into small groups, and hand out chart paper and copies of BLM 8.SS.4.1: Volume Problems, which presents a variety of volume problems.
3. Ask groups to record their answers to the volume problems on the chart paper. They must explain what method they chose for solving the problem, why they chose that method, and why they think their answer is reasonable.
4. Have groups take turns presenting their problems to the class. Provide opportunities for the other groups to ask questions and add to the responses.
5. Ask students to create and solve a new volume problem in their math journals.

Observation Checklist

☑ Use students' math journal responses to determine whether they can do the following:

- ❒ Communicate mathematically.
- ❒ Apply effective strategies to solve volume problems.

Connecting Surface Area and Volume in Real Life

Introduction:

Krispee Oats Cereal Company wants to save as much money as possible. In order to do this, the company wants to have a high volume of cereal in the box, but a low surface area to avoid wasting money on the cardboard packaging. Students will design a cereal box that can hold 8750 cm^3 of cereal.

Purpose:

Students will demonstrate a comprehensive understanding of surface area and volume of right rectangular prisms, right triangular prisms, and right cylinders.

Curricular Links: Art, ELA

Materials/Resources: Cardboard, card stock, or other sturdy paper, markers, BLM 5–8.21: Isometric Dot Paper

Optional Materials: Construction paper or other coloured paper, paint

Organization: Individual

Scenario:

- You work for Krispee Oats Cereal Company. Your job is to create a cereal box that will hold 8750 cm^3 of cereal but will have a low surface area, as the company is trying to keep costs down and does not want to spend a lot of money on the cardboard packaging.
- You must meet the following expectations:
  - Demonstrate, using isometric dot paper, at least three different designs of the cereal box showing all measurements, ensuring that the cereal box will hold 8750 cm^3 of cereal.
  - Determine the cost of each of your cereal boxes if the cardboard costs $0.50 per square centimetre.
  - Create a net, including measurements, of your chosen design.
  - Choose the design that best meets the criteria for your company.
  - Construct the cereal box.
  - Decorate the cereal box to make it attractive to the consumer.

Assessment:

The following rubric can be used to assess achievement of the mathematics learning outcomes.

Criteria	Meeting Expectations	Developing to Meet Expectations	Beginning to Meet Expectations	Incomplete
The student				
■ is able to determine the dimensions of prisms that will hold a particular volume	❒ uses understanding of volume to determine three different possible dimensions for a particular volume	❒ uses understanding of volume to determine two different possible dimensions for a particular volume	❒ uses understanding of volume to determine one possible dimension for a particular volume	❒ does not determine a possible dimension for a particular volume
■ demonstrates an understanding of calculating the surface area of a prism	❒ includes clear step-by-step procedures for calculating the surface area of a prism	❒ includes most steps for calculating the surface area of a prism	❒ includes few steps for calculating the surface area of a prism	❒ includes no steps for calculating the surface area of a prism
■ demonstrates how to solve problems involving surface area	❒ demonstrates a comprehensive understanding of problem solving involving surface area by including clear step-by-step procedures for calculating the cost of materials	❒ demonstrates an understanding of problem solving involving surface area by including the cost of materials using surface area	❒ demonstrates minimal understanding of problem solving involving surface area when attempting to include the cost of materials using surface area	❒ does not attempt to show an understanding of solving problems involving surface area
■ demonstrates an understanding of a net	❒ accurately draws and labels a net of a 3-D object that represents the final product	❒ draws and labels a net of a 3-D object that may or may not represent the final product	❒ draws and labels a net of a 3-D object but it does not represent the final product	❒ does not draw or label a net of a 3-D object

Extension:

What would be the volume and surface area of the cardboard box that could hold 20 boxes of your final product?

Shape and Space (Transformations)—8.SS.6

Enduring Understandings:

Many geometric properties and attributes of shapes are related to measurement.

Tessellations are created using transformations.

General Learning Outcome:

Describe and analyze position and motion of objects and shapes.

Specific Learning Outcome(s):	Achievement Indicators:
8.SS.6 Demonstrate an understanding of tessellations by ■ explaining the properties of shapes that make tessellating possible ■ creating tessellations ■ identifying tessellations in the environment [C, CN, PS, T, V]	➛ Identify in a set of regular polygons those shapes and combinations of shapes that will tessellate, and use angle measurements to justify choices. ➛ Identify in a set of irregular polygons those shapes and combinations of shapes that will tessellate, and use angle measurements to justify choices. ➛ Identify a translation, reflection, or rotation in a tessellation. ➛ Identify a combination of transformations in a tessellation. ➛ Create a tessellation using one or more 2-D shapes, and describe the tessellation in terms of transformations and conservation of area. ➛ Create a new tessellating shape (polygon or non-polygon) by transforming a portion of a tessellating polygon, and describe the resulting tessellation in terms of transformations and conservation of area. ➛ Identify and describe tessellations in the environment.

PRIOR KNOWLEDGE

Students may have had experience with the following:

- Identifying a single transformation (translation, rotation, or reflection) of 2-D shapes
- Demonstrating an understanding of angles by
  - identifying examples of angles in the environment
  - classifying angles according to their measure
  - estimating the measure of angles using 45°, 90°, and 180° as reference angles
  - determining angle measures in degrees
  - drawing and labelling angles when the measure is specified
- Describing and comparing the sides and angles of regular and irregular polygons
- Performing a combination of transformations (translations, rotations, or reflections) on a single 2-D shape, and drawing and describing the image
- Performing a combination of successive transformations of 2-D shapes to create a design, and identifying and describing the transformations
- Performing and describing transformations of a 2-D shape in all four quadrants of a Cartesian plane (limited to integral vertices)

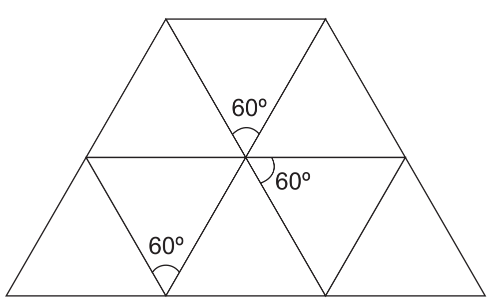
BACKGROUND INFORMATION

Tessellations

Three regular polygons—equilateral triangles, squares, and hexagons—will tessellate the plane because their angles are a factor of 360º. Irregular polygons whose angles add to a factor of 360º will also tessellate the plane. Polygons that tessellate the plane can be used to make tessellations.

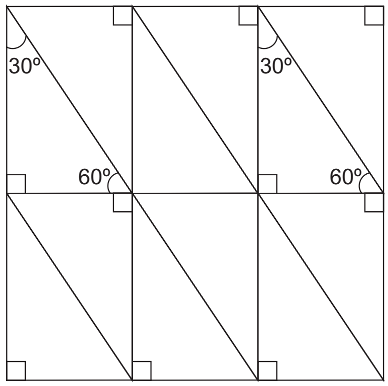
Examples:

Regular Polygon That Tessellates the Plane



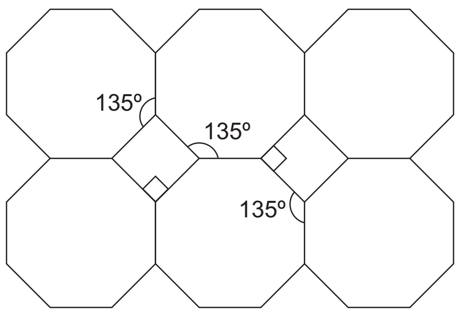
The angles of an equilateral triangle are each 60º. Therefore, no matter how you arrange the triangle, the shape will tessellate.

Irregular Polygon That Tessellates the Plane



The angles of every triangle add up to 180º (which is a factor of 360º). Therefore, all triangles should tessellate, although a little more manoeuvring may be needed if they are not equilateral triangles. The same is true of all quadrilaterals.

Combination of Polygons That Tessellate the Plane



Sometimes it may take a combination of regular and/or irregular polygons to create a tessellation. In this example, a regular octagon (with interior angle measure 135º) and a regular quadrilateral (with interior angle measure 90º) will tessellate, since the sum of the angles that meet at a point is (135º + 135º + 90º = 360º) a factor of 360º.

Polygons are transformed via translations (slides), reflections (flips), and rotations (turns) in order to tessellate the plane.

M. C. Escher is an artist famous for his work in tessellations. Escher-style tessellations begin with a regular polygon that tessellates. A portion of one side is removed and taped onto the opposite side to create a new shape that has the same area as the original shape. Inside the shape, creative images are filled in to make a work of art.

For more information, visit the following websites:

- The Official M. C. Escher Website. <www.mcescher.com/>.
- Tessellations.org. "Do-It-Yourself." <www.tessellations.org/>.

Note: These websites were current in June 2015. If they are not available, use a search engine with the key words "Escher" or "tessellations" to find other possible resources.

Mathematical Language

irregular polygon
quadrilaterals
reflection
regular polygon
rotation
tessellation
transformation
translation

Learning Experiences

Assessing Prior Knowledge

Materials: BLM 8.SS.6.1: Coordinate Image

Organization: Individual

Procedure:

1. Tell students that the purpose of this learning activity is to have them demonstrate their understanding of transformations.
2. Have students transform image ABCD on BLM 8.SS.6.1: Coordinate Image:
   - Translate ABCD [7, –2].
   - Reflect ABCD over the *x* axis.
   - Rotate ABCD 90° clockwise, with the origin as the point of rotation.
   - Locate ABCD in its new location using coordinates.

Observation Checklist

☑ Observe students' responses to determine whether they can do the following:
- ❒ Translate images.
- ❒ Reflect images.
- ❒ Rotate images.

Suggestions for Instruction

- **Identify in a set of regular polygons those shapes and combinations of shapes that will tessellate, and use angle measurements to justify choices.**
- **Identify in a set of irregular polygons those shapes and combinations of shapes that will tessellate, and use angle measurements to justify choices.**

Materials: Regular polygons (square, equilateral triangle, regular hexagon, regular pentagon, regular octagon) and irregular polygons (rhombus, rectangle, parallelogram, isosceles triangle, trapezoid, irregular pentagon), BLM 8.SS.6.2: Tessellating the Plane, math journals

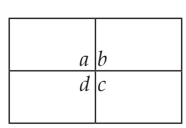
Organization: Small group/pairs/whole class/individual

Procedure:

1. Divide the class into small groups, and provide each group with a set of shapes. Using the Sort and Predict strategy, have students sort the shapes into the following two categories: regular polygons and irregular polygons.
2. Review the results and, as a class, generate the definitions of *regular polygons* and *irregular polygons*.
3. Using the regular hexagon, demonstrate how to tessellate the plane.
4. Use a regular pentagon to demonstrate that it does not tessellate the plane.
5. Using the Think-Pair-Share strategy, have students come up with the definition of *tessellating the plane*.
6. As a class, come up with a common definition of *tessellating the plane*.
7. Let the groups try to tessellate the rest of the shapes they have. Once they have done this, have students make a chart for the shapes, identifying each shape, noting the sum of the interior angles of the common vertices, and indicating whether or not the shapes tessellate the plane. (See BLM 8.SS.6.2: Tessellating the Plane.)

   Example:

   The following is a shape that tessellates. Four of these shapes create a tessellation. Angles *a*, *b*, *c*, and *d* form the interior angles of the common vertices. The sum of angles *a*, *b*, *c*, and *d* must equal 360° in order to tessellate.



8. Demonstrate to students how to measure the interior angles of the common vertices. Then have them measure the interior angles of the common vertices of those that tessellate the plane, as well as those that do not tessellate the plane.
9. Ask students:
   Why do some shapes tessellate the plane and others do not?
10. After discussing why some shapes tessellate the plane and others do not, have students answer the following questions in their math journals:
    - What are three regular polygons that tessellate the plane?
    - Why do they tessellate the plane?
    - What are three irregular polygons that tessellate the plane?
    - Why do they tessellate the plane?
    - Why don’t regular pentagons and regular octagons tessellate the plane?

Observation Checklist

☑ Observe students’ responses to determine whether they can do the following:

- ❒ Identify regular polygons that tessellate the plane.
- ❒ Identify irregular polygons that tessellate the plane.
- ❒ Explain why some polygons tessellate the plane and others do not.

Note: In their responses, students should indicate that only polygons whose interior angles measure 360^{o} or whose interior angles add to a factor of 360^{o} will tessellate the plane.

- **Identify in a set of regular polygons those shapes and combinations of shapes that will tessellate, and use angle measurements to justify choices.**
- **Identify in a set of irregular polygons those shapes and combinations of shapes that will tessellate, and use angle measurements to justify choices.**

Materials: BLM 5–8.21: Isometric Dot Paper, BLM 5–8.22: Dot Paper

Organization: Individual

Procedure:

1. Tell students that they will be creating tessellations using dot paper and that they will need to label their shapes and angles.
2. Have students create and label the following:
   - a tessellation using one regular polygon
   - a tessellation using one irregular polygon
   - a tessellation using two or more regular polygons
   - a tessellation using two or more irregular polygons

Observation Checklist

☑ Observe students' responses to determine whether they can do the following:

- ❒ Identify a regular polygon that tessellates.
- ❒ Identify a combination of regular polygons that tessellate.
- ❒ Identify an irregular polygon that tessellates.
- ❒ Identify a combination of irregular polygons that tessellate.

- **Identify a translation, reflection, or rotation in a tessellation.**
- **Identify a combination of transformations in a tessellation.**

Materials: BLM 8.SS.6.3: Tessellation Slideshow, BLM 8.SS.6.4: Tessellation Recording Sheet, 1 computer per small group, BLM 8.SS.6.5: Tessellation Transformation, math journals

Organization: Small group/whole class/individual

Procedure:

1. Tell students that they will view the 10 tessellation slides included in BLM 8.SS.6.3: Tessellation Slideshow. Hand out copies of BLM 8.SS.6.4: Tessellation Recording Sheet.
2. Divide the class into small groups. Ask each group to view the slideshow and decide which transformation was used for each of the 10 images, recording their decisions on the recording sheet provided.
3. As a class, review the slideshow and come to a consensus on which transformation was used for each tessellation.
4. Have students, individually, complete BLM 8.SS.6.5: Tessellation Transformation.

Observation Checklist

☑ Observe students’ responses to determine whether they can do the following:

- ❒ Identify a translation used in a tessellation.
- ❒ Identify a reflection used in a tessellation.
- ❒ Identify a rotation used in a tessellation.
- ❒ Identify a combination of transformations used in a tessellation.

- **Create a tessellation using one or more 2-D shapes, and describe the tessellation in terms of transformations and conservation of area.**
- **Create a new tessellating shape (polygon or non-polygon) by transforming a portion of a tessellating polygon, and describe the resulting tessellation in terms of transformations and conservation of area.**

Materials: BLM 5–8.9: Centimetre Grid Paper, white paper, scissors, math journals

Organization: Individual

Procedure:

1. Ask students to list the various shapes that tessellate.
2. Hand out copies of BLM 5–8.9: Centimetre Grid Paper, and have students, individually, draw a shape of their choice that they know will tessellate.
3. Have students determine the area of the shape, and then cut out the shape.
4. Instruct students to draw the shape on the white paper provided. Have students choose a transformation that works with their shape and transform the shape to tessellate the plane.
5. Ask students to determine the area of the newly tessellated shapes.
6. Ask students to answer the following questions:
   - What do you notice when you compare the area of the original shape and the area of the new shape?
   - What statement can be made about the area of the tessellating shapes?
7. Ask students to explain, in their math journals, why the area of a tessellating shape stays the same.

Observation Checklist

☑ Observe students' responses to determine whether they can do the following:

- ❒ Describe a tessellation using transformations.
- ❒ Reason why area is conserved in a tessellation.

- **Identify and describe tessellations in the environment.**

Materials: Paper and clipboard

Organization: Pairs or small groups

Procedure:

1. Tell students that they will be working in a small group to create a tessellation scavenger hunt and then exchange clues with another group. Give them the following parameters:
   - You must have between eight and ten points of interest.
   - A point of interest is a tessellation to which you want to draw attention.
   - All your points of interest must be located within the school grounds.
   - You need to provide a neat copy of your clues so that the other group can find your points of interest without difficulty.
   - You need to provide an answer key with a sketch of the tessellation and a clear description of its location.
2. Have each group exchange clues with another group and try to find the other group's tessellation points of interest.
3. Have students meet up with the group they exchanged with to make sure both groups have found all the tessellation points of interest.

Observation Checklist

☑ Observe students' responses to determine whether they can do the following:

- ❒ Identify and describe tessellations in the environment.

Escher Tessellations

Introduction:

This task allows students to research M. C. Escher and then create a piece of artwork that represents his use of tessellations.

Purpose:

Students will use the following skills: transformations to create tessellations, principles and elements of art, the inquiry process.

Curricular Links: Mathematics, Art, ELA, LwICT

Materials/Resources: Internet access, various art media (e.g., pencil crayons, paint, charcoal—depending on the comfort level of the teacher), manila tag paper to use for the template, white paper for the end project

Organization: Individual/small group

Scenario:

You are going to research M. C. Escher and identify how he used geometric shapes to create amazing works of art. Then, using his techniques, you will create your own Escher-like tessellation for display in the classroom or in the school hallway.

You will prepare a report on your research findings about Escher. You will explain how you used geometric shapes to create your tessellation and what transformations you used. You can choose the format of your report from the following options: a written report, a brochure, or a presentation (e.g., using *PowerPoint*, *Photo Story*, or *Movie Maker*).

Assessment:

The following rubric can be used to assess achievement of the mathematics learning outcomes.

Note: Other rubrics may be added to assess Art, ELA, and LwICT learning outcomes.

Criteria	Meeting Expectations	Developing to Meet Expectations	Beginning to Meet Expectations	Incomplete
The student				
■ demonstrates an understanding of tessellation by creating tessellations	❐ provides a creative tessellation that clearly models Escher's work	❐ provides a tessellation that models Escher's work	❐ provides a tessellation that somewhat models Escher's work	❐ does not provide a tessellation
■ demonstrates an understanding of tessellation by explaining the properties of shapes that make tessellating possible	❐ provides a clear explanation of how geometric shapes were used to create the tessellation, including reference to interior angles	❐ provides a general explanation of how geometric shapes were used to create the tessellation	❐ provides a vague or minimal explanation of how geometric shapes were used to create the tessellation	❐ provides no explanation of how geometric shapes were used to create the tessellation
	❐ provides a clear explanation of what transformations were used to create the tessellation	❐ provides a general explanation of what transformations were used to create the tessellation	❐ provides a vague or minimal explanation of what transformations were used to create the tessellation	❐ provides no explanation of what transformations were used to create the tessellation

Grade 8 Mathematics

Statistics and Probability

Statistics and Probability (Data Analysis)—8.SP.1

Enduring Understanding:

Data are gathered and organized in different ways, which may have an impact on what the data display.

General Learning Outcome:

Collect, display, and analyze data to solve problems.

Specific Learning Outcome(s):	Achievement Indicators:
8.SP.1 Critique ways in which data are presented. [C, R, T, V]	➢ Compare the information that is provided for the same data set by a set of graphs, such as circle graphs, line graphs, bar graphs, double bar graphs, or pictographs, to determine the strengths and limitations of each graph. ➢ Identify the advantages and disadvantages of different graphs, such as circle graphs, line graphs, bar graphs, double bar graphs, or pictographs, in representing a specific set of data. ➢ Justify the choice of a graphical representation for a situation and its corresponding data set. ➢ Explain how a formatting choice, such as the size of the intervals, the width of bars, or the visual representation, may lead to misinterpretation of the data. ➢ Identify conclusions that are inconsistent with a data set or graph, and explain the misinterpretation.

Prior Knowledge

Students may have had experience with the following:

- Constructing and interpreting concrete graphs and pictographs to solve problems
- Constructing, labelling, and interpreting bar graphs to solve problems
- Constructing and interpreting pictographs and bar graphs involving many-to-one correspondence to draw conclusions
- Constructing and interpreting double bar graphs to draw conclusions
- Creating, labelling, and interpreting line graphs to draw conclusions
- Graphing collected data and analyzing the graph to solve problems
- Demonstrating an understanding of central tendency and range by
  - determining the measures of central tendency (mean, median, mode) and range
  - determining the most appropriate measures of central tendency to report findings
- Determining the effect on the mean, median, and mode when an outlier is included in a data set
- Constructing, labelling, and interpreting circle graphs to solve problems

Background Information

The way data are organized and represented may have an impact on what one might interpret from the data. Graphs are a common way of organizing, representing, and communicating information. Different graphs are used, depending on what information is being represented and communicated.

Types of Graphs

Graphs are used to provide visual displays of data. It is important to know what type of data have been collected and the information that is to be communicated before deciding on what type of graph to use. Middle Years students need to have a good understanding of the advantages and disadvantages of each type of graph. This understanding will enable them to determine the appropriate graph for their data and to defend their choices.

A description of the various types of graphs is provided below, followed by a chart outlining the advantages and disadvantages of different graphs.

- **Bar Graphs**

  Bar graphs usually compare frequency of discrete data. Therefore, there are spaces between bars. Bars can be drawn either vertically or horizontally.

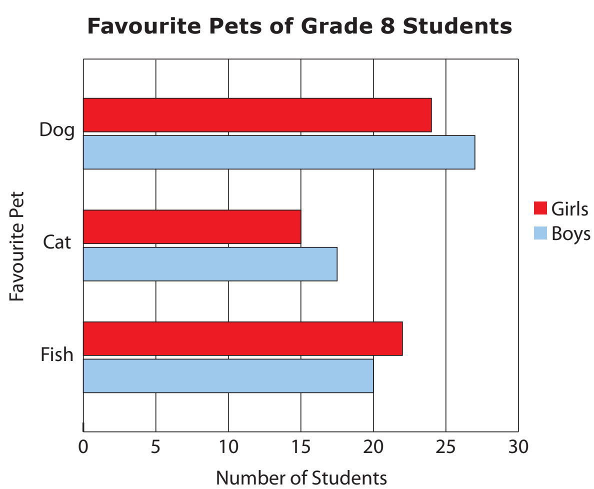
  Example:

  Note: If part of the scale is omitted, then a squiggle in the vertical axis is used. However, this tends to give a misleading visual picture.

- **Double Bar Graphs**

  These types of graphs use pairs of bars to make comparisons between and among sets of data. Bars can be vertical or horizontal.

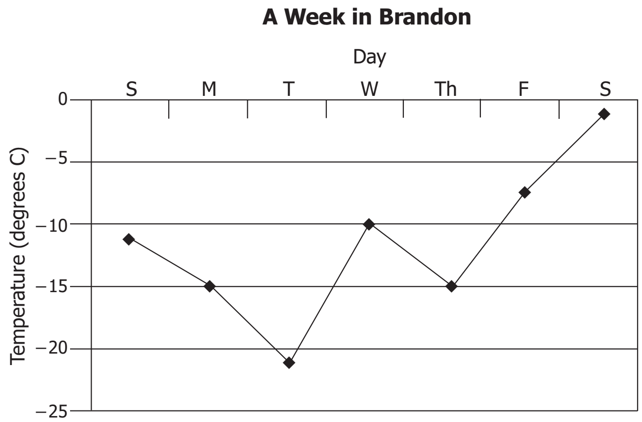
  Example:



- **Line Graphs (Broken Line Graphs)**

These types of graphs are appropriate for indicating trends or relationships and are used primarily to show a quantity changing over time (e.g., temperature change over 24 hours, average monthly rainfall, yearly school enrolments). For example, a survey of the favourite seasons of a Grade 8 class could not be put on a line graph, as the data do not involve change over time.

Example:



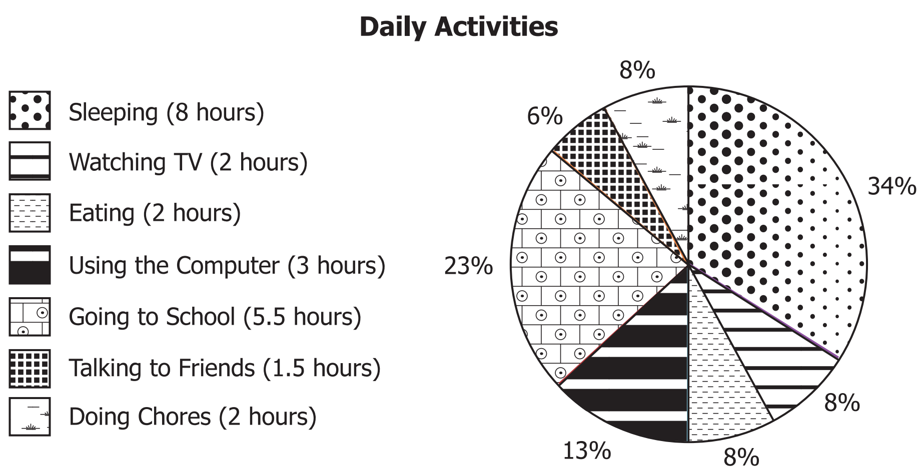
- **Circle Graphs (Pie Graphs)**

In circle graphs, the data are represented by sectors (parts) of a circle (whole); the total of all the sectors should be 100% of the data. Each section of the circle represents a part or percentage of the whole.

Circle graphs

- show the ratio of each part to the whole, not quantities
- are almost always made from data converted to percentages of the total
- show ratios—therefore, comparisons can be made between different-sized quantities (e.g., results from the class survey can be compared to results from a whole school survey)

Example:



Advantages and Disadvantages of Graphs			
Graphs	Purpose(s)	Advantage(s)	Disadvantage(s)
Bar graphs	■ compare frequency of data (usually discrete)	■ are easy to read and interpret ■ can be used to compare two or more related sets of data	■ can be misleading if part of the scale along one axis is compressed
Line graphs	■ show changes in a single variable over time	■ can be used to observe changes over time ■ can be used to find individual pieces of data	■ can be used only if data change over time ■ can be misleading if part of the scale along one axis is compressed
Circle graphs	■ compare groups of data to the whole set of data	■ can be used to see the ratio of each part to the whole group	■ cannot retrieve individual pieces of data because data are grouped

Mathematical Language

bar graph
circle graph
distort
double bar graph
double line graph
interval
line graph
pictograph
trend

LEARNING EXPERIENCES

Assessing Prior Knowledge

Materials: BLM 8.SP.1.1: Data Analysis Pre-Assessment

Organization: Individual

Procedure:

1. Tell students that they will be extending their understanding of data analysis over the next few lessons; however, you first need to find out what they already know about graphs.
2. Provide students with copies of BLM 8.SP.1.1: Data Analysis Pre-Assessment.
3. Have students complete the sheet individually.
4. Students will complete BLM 8.SP.1.1 at the end of the unit as a post-assessment.

Observation Checklist

☑ Observe students' responses to determine whether they can do the following:

- ❒ Recognize and explain why a broken line graph will not be the best way to represent which search engines students prefer.
- ❒ Understand that a circle graph must be filled completely.
- ❒ Recognize and explain why 25% of a circle graph is not 60°.
- ❒ Recognize and explain that the intervals on the sample bar graph are not properly spaced (spaces are unequal, graph does not start at zero).
- ❒ Correctly interpret a circle graph.

Note: Students will be creating and reading graphs in social studies and science. Linking graphs to those subject areas would be beneficial.

Suggestions for Instruction

- **Compare the information that is provided for the same data set by a set of graphs, such as circle graphs, line graphs, bar graphs, double bar graphs, or pictographs, to determine the strengths and limitations of each graph.**
- **Identify the advantages and disadvantages of different graphs, such as circle graphs, line graphs, bar graphs, double bar graphs, or pictographs, in representing a specific set of data.**
- **Justify the choice of a graphical representation for a situation and its corresponding data set.**

Materials: Graphing software or white paper, graph paper, rulers, BLM 5–8.23: Understanding Words Chart, BLM 8.SP.1.2: Data, math journals, BLM 8.SP.1.1: Data Analysis Pre-Assessment

Organization: Small group/whole class/individual

Procedure:

1. Tell students that, through exploration, they will be able to determine the strengths and limitations of different graphs, identify advantages and disadvantages of graphs, and justify their choice of graphical representation for a given situation.
2. Use BLM 5–8.23: Understanding Words Chart to address the key vocabulary for this unit.
3. Divide the class into small groups, and provide each group with a copy of BLM 8.SP.1.2: Data. Ask each group to create a bar graph, a line graph, and a circle graph for each set of data presented.
4. Each group will discuss the following questions in relation to the data and present their findings to the class:
   - Based on your data, what are the strengths and limitations of each of the graphs you made?
   - What are the advantages and disadvantages of each type of graph?
   - Which graph do you feel is best for representing each set of data provided? Why?
5. As the groups present their information, record responses to each question in a chart such as the following. By the end, you will have generated a variety of strengths and limitations, advantages and disadvantages, and best choice of graph per set of data.

Strengths and Limitations	Advantages and Disadvantages	Best Choice of Graph

6. Ask students to give an example, in their math journals, of the type of data that could be collected that would best be portrayed in a bar graph, a broken line graph, and a circle graph.
7. Have students complete BLM 8.SP.1.1: Data Analysis Pre-Assessment as a post-assessment.

Observation Checklist

☑ Observe students' responses to determine whether they can do the following:

- ❒ Demonstrate an understanding of strengths and weakness of each type of graph.
- ❒ Complete BLM 8.SP.1.1 as a post-assessment.

Suggestions for Instruction

- **Explain how a formatting choice, such as the size of the intervals, the width of bars, or the visual representation, may lead to misinterpretation of the data.**
- **Identify conclusions that are inconsistent with a data set or graph, and explain the misinterpretation.**

Materials: BLM 8.SP.1.3: Graph Samples, math journals

Organization: Small group/individual

Procedure:

1. Tell students that they will be analyzing different sets of graphs to determine which graph best represents the given data and explaining why one graph leads to a misinterpretation of the data represented.
2. Hand out graph **Sample 1**, **Sample 2**, and **Sample 3** from BLM 8.SP.1.3: Graph Samples, one at a time.
3. For **each** sample, have students, working in groups, discuss the following:
   - What can be said about these graphs?
   - What scenario do the data display?
   - Is there something that can be done to each graph to make it clearer? Explain.
   - Do the graphs display the same or different data? Explain.
   - What are the advantages and disadvantages of each graph?
   - Which graph is more accurate? Explain.
   - Can either of the graphs be misinterpreted? Explain.
4. Have students explain, in their math journals, how the format of graphs (how the graphs are made) can lead to a misinterpretation of the data. Ask them to use words and diagrams to explain their thoughts.

Observation Checklist

☑ Observe students' responses to determine whether they can do the following:

- ❒ Explain how the format of graphs can lead to a misinterpretation of the data.

Statistics and Probability (Chance and Uncertainty)—8.SP.2

Enduring Understandings:

The principles of probability of a single event also apply to independent events.

Probability can be expressed as a fraction or decimal between 0 and 1, where 0 indicates an impossible event and 1 indicates a certain event.

Probabilities can be expressed as ratios, fractions, percents, and decimals.

General Learning Outcome:

Use experimental or theoretical probabilities to represent and solve problems involving uncertainty.

Specific Learning Outcome(s):	Achievement Indicators:
8.SP.2 Solve problems involving the probability of independent events. [C, CN, PS, T]	➛ Determine the probability of two independent events and verify the probability using a different strategy. ➛ Generalize and apply a rule for determining the probability of independent events. ➛ Solve a problem that involves determining the probability of independent events.

Prior Knowledge

Students may have had experience with the following:

- Describing the likelihood of a single outcome occurring, using words such as
  - impossible
  - possible
  - certain
- Comparing the likelihood of two possible outcomes occurring, using words such as
  - less likely
  - equally likely
  - more likely

- Demonstrating an understanding of probability by
  - identifying all possible outcomes of a probability experiment
  - differentiating between experimental and theoretical probability
  - determining the theoretical probability of outcomes in a probability experiment
  - determining the experimental probability of outcomes in a probability experiment
  - comparing experimental results with the theoretical probability for an experiment
- Expressing probabilities as ratios, fractions, and percents
- Identifying the sample space (where the combined sample space has 36 or fewer elements) for a probability experiment involving two independent events
- Conducting a probability experiment to compare the theoretical probability (determined using a tree diagram, table, or another graphic organizer) and experimental probability of two independent events

Background Information

Probability

Probability refers to the chance of an event occurring. The probability of an event must be greater than or equal to 0 and less than or equal to 1.

Note: Students in Grade 8 will be working only with the probabilities of independent events.

Definitions

independent events

Events in which the theoretical probability of an event occurring does not depend on the results of another event.

Example:

Rolling a number cube and then selecting a card from a deck.

What is the probability of rolling a 6 on a number cube and then pulling a 6 from a deck of cards?

$\text{P(6 cube)} = \frac{1}{6}$ $\text{P(6 card)} = \frac{1}{13}$ so $\text{P(6 cube and 6 card)} = \frac{1}{78}$

dependent events

Events in which the theoretical probability of an event occurring does depend on the results of another event.

Example:

Selecting a red card from a deck and then selecting the Queen of Clubs without putting the first card back.

$P(\text{red}) = \frac{1}{2}$ $P(\text{Queen of Clubs}) = \frac{1}{51}$ so $P(\text{red, then Queen of Clubs}) = \frac{1}{102}$

Organizing Outcomes/Results

There are different strategies for organizing favourable outcomes, such as tables and tree diagrams.

Example:

If Joe has six cards numbered 1 to 6 and a regular six-sided number cube, what is the probability of turning a 1 and rolling a 1 at the same time?

This scenario can be written as follows: What is P(1,1)?

Table

		Number Cube					
		1	2	3	4	5	6
Card	1	1,1	1,2	1,3	1,4	1,5	1,6
	2	2,1	2,2	2,3	2,4	2,5	2,6
	3	3,1	3,2	3,3	3,4	3,5	3,6
	4	4,1	4,2	4,3	4,4	4,5	4,6
	5	5,1	5,2	5,3	5,4	5,5	5,6
	6	6,1	6,2	6,3	6,4	6,5	6,6

$$P(\text{event}) = \frac{\textit{favourable outcome}}{\textit{total number of outcomes}}$$

$$P(1,1) = \frac{1}{36}$$

Tree Diagram

Card	Cube	Outcome
1	1	1,1
	2	1,2
	3	1,3
	4	1,4
	5	1,5
	6	1,6
2	1	2,1
	2	2,2
	3	2,3
	4	2,4
	5	2,5
	6	2,6
3	1	3,1
	2	3,2
	3	3,3
	4	3,4
	5	3,5
	6	3,6
4	1	4,1
	2	4,2
	3	4,3
	4	4,4
	5	4,5
	6	4,6
5	1	5,1
	2	5,2
	3	5,3
	4	5,4
	5	5,5
	6	5,6
6	1	6,1
	2	6,2
	3	6,3
	4	6,4
	5	6,5
	6	6,6

$P(1,1) = \frac{1}{36}$ This can also be expressed as 1:36, ≈3% or ≈ 0.03.

Mathematical Language

certain
experimental probability
impossible
independent events
less likely
likely
more likely
outcome
probability
probable
simulation
theoretical probability

Learning Experiences

Assessing Prior Knowledge

Materials: BLM 8.SP.2.1: Probability Pre-Assessment

Organization: Individual

Procedure:

1. Tell students that they will be extending their understanding of probability over the next few lessons; however, you first need to find out what they already know about probability.
2. Hand out copies of BLM 8.SP.2.1: Probability Pre-Assessment.
3. Have students complete the pre-assessment individually.

Observation Checklist

☑ Observe students' responses to determine whether they can do the following:

- ❒ Determine all possible outcomes of a specified event.
- ❒ Use an organization method to organize the outcomes.
- ❒ Determine the probability of a specified event.
- ❒ Represent probability as a ratio, fraction, decimal, and percent.

- **Determine the probability of two independent events and verify the probabilities using a different strategy.**

Materials: 1 six-sided number cube and 1 coin per group, BLM 5–8.23: Understanding Words Chart, BLM 8.SP.2.2: Tree Diagram

Organization: Individual/small group/whole class

Procedure:

1. Provide each student with a copy of BLM 5–8.23: Understanding Words Chart, and have students explore their understanding of key mathematical terms.
2. Divide the class into small groups, and present students with the following scenario:

   John rolls a six-sided number cube at the same time that Sue flips a coin. Determine all possible outcomes if they complete the task at the exact same time. (Provide each group with a number cube and a coin in case they need to manipulate the items to help them determine the outcomes.)
3. Allow groups to organize their work as they see fit. When it is time to review their work, write their outcomes on the whiteboard using a tree diagram.
4. When all possible outcomes are recorded, explain to students how a tree diagram allows favourable outcomes to be determined in an organized manner.
5. Ask students to state P(3, H), P(odd, T), P(1 or 2, H).
6. Have students complete BLM 8.SP.2.2: Tree Diagram.

Observation Checklist

☑ Observe students' responses to determine whether they can do the following:

- ❒ Successfully use a tree diagram to determine probabilities.

Suggestions for Instruction

- **Determine the probability of two independent events and verify the probabilities using a different strategy.**

Materials: A King, Queen, and Jack of Spades, and a 1, 2, and 3 of Hearts of a deck of cards per group, BLM 8.SP.2.3: Table

Organization: Small group/whole class

Procedure:

1. Divide the class into small groups, and present students with the following scenario:

   Rita has the Jack, Queen, and King of Spades from a deck of cards, and Jessica has the 1, 2, and 3 of Hearts from the deck of cards. If they both flip a card at the same time, what are all the possible outcomes? (Provide each group with the Jack, Queen, and King of Spades and the 1, 2, and 3 of Hearts from a deck of cards in case they need to manipulate items to help them determine the outcomes.)
2. Tell students that their task is to determine all possible outcomes using a strategy for organizing the outcomes other than a tree diagram.
3. Have each group present their method to the class. If no one demonstrates how a table could be used, you will need to demonstrate it.
4. Ask students to state P(J, 1), P(face card, odd), P(Q, 1).
5. Have students complete BLM 8.SP.2.3: Table.

Observation Checklist

☑ Observe students’ responses to determine whether they can do the following:

- ❒ Successfully use a table to determine probabilities.

- **Generalize and apply a rule for determining the probability of independent events.**
- **Solve a problem that involves determining the probability of independent events.**

Materials: BLM 8.SP.2.4: Probability Problems, BLM 8.SP.2.5: Probability Problem Practice, chart paper

Organization: Small group/whole class

Procedure:

1. Divide students into small groups, and provide each group with one problem from BLM 8.SP.2.4: Probability Problems, as well as chart paper. Ask groups to solve their respective problems and be prepared to present their solutions.
2. As each group presents its problem and solution strategy to the class, allow other groups to ask questions and add to the solution.
3. Have students identify the various strategies that the groups used to solve the problems.
4. BLM 8.SP.2.5: Probability Problem Practice provides additional problems for practice. Discuss rules for finding the probability of independent events. (Students should generalize that multiplication can be used to find solutions to the probability questions.)

Observation Checklist

☑ Observe students' responses to determine whether they can do the following:

- ❒ Generalize a rule to determine the number of outcomes for two independent events.
- ❒ Apply a generalized rule and knowledge about probability in order to reason mathematically.
- ❒ Determine the possible outcomes of a probability experiment involving two independent events.
- ❒ Determine the probabilities of favourable outcomes.

Grade 8 Mathematics

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