

Grade 12 Introduction to Calculus and Advanced Mathematics

Course Code 3908

Course Credit 1.0

ENGLISH Program

Discipline Overview

The Kindergarten to Grade 12 mathematics curriculum in the English Program is designed to meet learners' interests, skills, and needs so they can realize that mathematics is a way of building their understanding of the world and that it is part of their everyday lives.

The learning outcomes of this program of study are divided into four areas:

- Number
- Patterns and relations
- Shape and space
- Statistics (starting in Grade 2) and probability (starting in Grade 5)

These areas reflect the nature of mathematics from Kindergarten to Grade 12.

The study of mathematics promotes the development of global competencies and supports learners' cultivation of enduring understandings. It also fosters the development of logical thinking, problem-solving, and data analysis skills.

Learning experiences and evaluation in the mathematics classroom are based on a problem-based learning approach that allows learners to make connections between their conceptual understanding and various mathematical processes. Integrating these processes into learning helps learners to understand the nature of mathematics, and to make sense of it so they can learn and use mathematics in and out of school throughout their lives.

Course Overview

Grade 12 Introduction to Calculus and Advanced Mathematics learners will study four topics with specific content outcomes. The topics are limits, derivatives, application of derivatives, and integration. They will also study any four chosen topics from a list of thirteen—seven of which have specific learning outcomes. The core topics are complex numbers and polar coordinates, statistics, number theory, matrices and systems of equations, 3-dimensional geometry, vectors, and conic sections. The additional topics are fractal geometry, calculus topics to extend beyond

Introduction to Calculus content, history of mathematics, applications of mathematics to computer science and combinatorics extending beyond permutations and combinations, and interdisciplinary project. The flexibility of the course allows teachers to choose the topics and encourages the input of learners.

Global Competencies in Mathematics



Critical Thinking

Critical thinking in mathematics involves the ability to compare, evaluate, critique, justify, test, and validate ideas, representations, plans, or solutions using logical arguments, criteria, and evidence. It requires metacognition in learners, enabling them to solve mathematical problems and situations, communicate their reasoning effectively, and make ethical decisions.

- Learners research, use, and think about a variety of ideas and information strategically, efficiently, and effectively to make decisions and choices.
- Learners evaluate their own and others' ideas, as well as possible solutions, by considering different perspectives, biases, and the validity and relevance of supporting sources.
- Learners use inductive reasoning to explore and record results; to analyze mathematical ideas, problems, and situations; to make observations and generalizations from patterns; and to test these generalizations based on criteria and evidence.
- Learners recognize that certain math beliefs influence how they perceive themselves as math learners.
- Learners demonstrate a willingness to reconsider their own thinking and to consider others' thinking about mathematical ideas, problems, or situations.
- Learners ask relevant and clarifying questions to further learning and enhance comprehension of mathematical ideas, concepts, problems, and situations.
- Learners make judgments based on thoughtful criteria to then make decisions and solve mathematical problems and situations, enabling them to take action in an informed manner.
- Learners use deductive reasoning to solve mathematical problems and situations, reach new conclusions based on what is already known or assumed to be true, and make ethical decisions.



Creativity

Creativity in mathematics involves flexible thinking, curiosity, and risk taking, as well as making connections to prior knowledge among learners; this allows learners to come up with innovative solutions to a variety of mathematical problems and situations by considering them from a new angle or by formulating new hypotheses.

- Learners embrace a learning environment of trust and respect that encourages them to make choices, take risks, and think flexibly—allowing them to make decisions and take action.
- Learners wonder, ask questions, and contemplate different mathematical ideas and concepts.
- Learners solve mathematical problems and situations using different ways to arrive at innovative solutions.
- Learners enrich and refine their reasoning by considering others' ideas.
- Learners formulate, adjust, and refine their plans for solving mathematical problems and situations by looking at them from a new angle.
- Learners validate and adapt plans, ideas, strategies, or solutions, while persevering through obstacles, so they can improve at solving mathematical problems and situations.
- Learners seek and use feedback from others to develop and consolidate their conceptual understanding, deepen their reasoning, and reflect on their processes for solving mathematical problems and situations.



Citizenship

Citizenship in mathematics involves the development of mathematical literacy that enables the application of mathematical ideas and concepts in a variety of everyday contexts, awakening learners' curiosity about their role as citizens who can actively contribute to society, think critically about the world, make informed decisions, and generate solutions to an issue from a variety of perspectives.

- Learners use mathematics as a means of developing their understanding of a range of complex social, cultural, economic, and political issues, and to help them reflect on them.
- Learners mobilize their mathematical knowledge and skills to analyze and understand issues related to discrimination, equity, and human rights by investigating or proposing solutions to a variety of mathematical problems or situations related to these issues.
- Learners mobilize their mathematical knowledge and skills to explore, analyze, and understand the impact of the interconnectedness of self, others, and the natural world by investigating or proposing solutions to a variety of mathematical problems and situations related to this issue.

- Learners show interest in others' approaches to mathematics and to different points of view, experiences, and worldviews, allowing them to better understand and solve mathematical problems and situations.
- Learners empathize with others whose ideas are different from their own and appreciate solutions to mathematical problems or situations proposed by others.
- Learners interact and learn with others in person or online in a responsible, respectful, and inclusive manner by welcoming and valuing diverse viewpoints, and by considering a range of ideas and perspectives when contributing to mathematical exchanges.
- Learners realize that their mathematical knowledge and skills will serve not only to improve their own quality of life but also that of others.
- Learners engage in meaningful mathematical inquiries, individually and in collaboration, in which they ask themselves and others questions so they can find equitable solutions and make ethical decisions.
- Learners appreciate how mathematics can be used to make and justify ethical decisions that lead to responsible and sustainable actions that affect themselves, their community, and the world.



Connection to Self

Connection to self in mathematics involves the learner's belief in their ability to approach and complete tasks, solve mathematical problems and situations, and persevere in the face of mathematical challenges. It also involves the learner's ability to engage positively in reflective practices about their learning in order to set goals for self-improvement.

- Learners believe in their ability to learn and understand the world of mathematics and its impact on their daily lives.
- Learners recognize the elements that shape their identity as math learners, and they see themselves as mathematicians.
- Learners allow themselves the time they need, and they implement strategies that foster a growth mindset to develop a positive relationship with mathematics.
- Learners consider reflecting on their own decisions, the efforts they deploy, the experiences they have, and feedback from others as learning opportunities to improve their knowledge and skills in mathematics.
- Learners reflect on their mathematical learning to set goals and make informed decisions that affect their well-being.

- Learners believe that their ability to learn, their talents, and their skills in mathematics will continue to improve throughout their lives through their hard work, perseverance, and effort.
- Learners are willing to take risks, ask for help, and persevere, despite obstacles.
- Learners demonstrate the ability to make changes and adapt to new mathematical contexts, knowing that they will learn from their mistakes and build on their personal strengths.
- Learners develop their autonomy, value their voice, and commit to their role in becoming lifelong mathematics learners.



Collaboration

Collaboration in mathematics involves adhering to a culture of exchanging ideas and viewpoints in order to improve, both collectively and individually, and to learn from and with others to develop and apply new ideas in mathematics.

- Learners collaborate with others, value diverse points of view, and consider a range of ideas and perspectives when contributing to mathematical exchanges.
- Learners participate actively and fully in learning experiences by sharing thinking and learning strategies with others to confirm or extend understandings of mathematical ideas; they respectfully voice their opinions, ideas, and conjectures.
- Learners value the contributions of others, making room for different points of view that will foster mathematical exchanges.
- Learners practise active listening, question their own and others' mathematical ways of thinking, and ask questions of others to deepen their understanding of mathematical concepts and ideas.
- Learners show a willingness to compromise and change their opinions when presented with convincing arguments during mathematical exchanges.
- Learners make sense of mathematical concepts and ideas by co-constructing their understanding with others.
- Learners support others and take responsibility for their roles throughout the learning process and in the execution of mathematical tasks.



Communication

Communication in mathematics involves the learners' ability to share their mathematical ideas, reasoning, and solutions in a variety of ways, including orally, in writing, concretely, graphically, and symbolically, and in various contexts. It enables learners to clarify and validate their ideas and reasoning, and to challenge their attitudes and beliefs about mathematics.

- Learners express their mathematical ideas and emotions about mathematics, taking into account non-verbal cues and adjusting what they say according to the context.
- Learners present their mathematical ideas visually, orally, in writing, graphically, or symbolically, taking into account the conventions related to the mode of communication used, their audience, and the types of communication contexts, while using clear, precise mathematical language.
- Learners understand how their words and actions shape their identity as mathematical learners and shape their relationships with others.
- Learners look for oral, non-verbal, or visual cues during exchanges to improve their understanding of terminology, what others are saying, ideas presented, and various solutions to mathematical problems and situations.
- Learners seek to understand different points of view and different solutions to a mathematical problem or situation by observing, practising active listening, and asking clarifying questions, thereby creating a culture of mutual communication.
- Learners recognize and accept that the ways they learn and represent their understanding may be different from those of others.
- Learners make sense of mathematical ideas, problems, and situations, and deepen their understanding, by making connections among their own language, mathematical terminology, and mathematical conventions.
- Learners contribute to mathematical exchanges and express their thoughts and emotions about mathematical ideas in a positive and respectful way, whether in person or online.
- Learners defend their points of view and their mathematical reasoning while accepting the points of view and reasoning of others in a constructive and responsible way; they understand how these exchanges benefit themselves as much as they do other members of their learning community.



Enduring Understandings

Mathematics, a tool for growth

Developing knowledge and skills in mathematics allows for the understanding of processes, theorems, concepts, situations, and their applications. Building mathematical knowledge allows learners to identify as citizens who believe in their ability to complete a task, a lesson, or a challenge successfully—motivating them to take action and persevere throughout their lifetime to achieve their goals.

Mathematics, a tool for seeing things differently

Mathematics is a part of the living world. It develops one's capacity to think fluidly and creatively, and to understand, interpret, and logically represent various phenomena that can be imperceptible or abstract, to put these phenomena in perspective regarding one another, and to analyze them from a different point of view.

Mathematics, a vital tool for understanding the world

Mathematics contributes to the analysis, the comprehension, the interpretation, and the description of the world in which we live. It allows one to study quantities, orders, spaces, numbers, and figures, and the links that exist among them.

Mathematics, an essential interdisciplinary tool for advancing society

Mathematics plays an important role in people's quest for innovation and solutions for the various problems encountered in advancing society. It is used in other disciplines to understand, describe, and interpret the phenomena that surround us, and it allows researchers to improve quality of life through innovation and the development of new technology.

Mathematics, an essential tool for all aspects of daily life

Mathematics is everywhere in society. Mathematical knowledge and skills are necessary for critically analyzing information in diverse social, personal, economic, political, cultural, and environmental contexts in order to make informed, ethical, and sustainable decisions in all aspects of daily life.



Learning Outcomes

Introduction to Calculus

Topic: Limits

- 1C.1.1.** Demonstrate an understanding of the concept of the limit.
- 1C.1.2.** Evaluate limits to analyze functions.
- 1C.1.3.** Apply the concept of limit to the continuity of a function.

Topic: Derivatives

- 1C.2.1.** Develop the definition of the derivative as the slope of a curve at a point.
- 1C.2.2.** Develop and apply differentiation rules.
- 1C.2.3.** Demonstrate an understanding of implicit differentiation.

Topic: Applications of Derivatives

- 1C.3.1.** Apply derivatives to solve problems involving the motion of particles.
- 1C.3.2.** Determine features of a function using derivatives to sketch the function accurately.
- 1C.3.3.** Apply derivatives to solve optimization and related rates problems.

Topic: Integrals

- 1C.4.1.** Demonstrate an understanding of the relationship between anti-differentiation and integration of functions.
- 1C.4.2.** Apply integration to solve problems.
- 1C.4.3.** Demonstrate and apply an understanding of the definite integral.

Grade 12 Advanced Mathematics

Topic: Complex Numbers and Polar Coordinates

- AM.1.1.** Define and perform operations on complex numbers.
- AM.1.2.** Make connections between complex numbers and quadratic equation solutions.
- AM.1.3.** Demonstrate an understanding of polar coordinates and their graphs.
- AM.1.4.** Make connections between complex numbers and polar coordinates.



Topic: Statistics

- AM.2.1.** Demonstrate an understanding of the concepts of measures of central tendency and spread.
- AM.2.2.** Demonstrate an understanding of probability distributions including the binomial distribution.
- AM.2.3.** Develop and apply the properties of a normal distribution.

Topic: Number Theory

- AM.3.1.** Apply proof techniques to prove mathematical theorems or statements.
- AM.3.2.** Explore, develop, and apply the properties of integers.
- AM.3.3.** Represent numbers in different bases.

Topic: Matrices and Systems of Equations

- AM.4.1.** Demonstrate an understanding of matrices.
- AM.4.2.** Perform operations on matrices.
- AM.4.3.** Solve systems of equations using matrices.

Topic: 3-Dimensional Geometry

- AM.5.1.** Demonstrate an understanding of 3-space.
- AM.5.2.** Represent and analyze lines, planes, and surfaces algebraically and graphically in 3-space.

Topic: Vectors

- AM.6.1.** Develop an understanding of vectors and perform basic vector operations.
- AM.6.2.** Demonstrate an understanding of the dot product and cross product of vectors to solve problems.
- AM.6.3.** Develop and apply the vector equation of a line.

Topic: Conic Sections

- AM.7.1.** Represent and analyze conic sections algebraically and geometrically.
- AM.7.2.** Demonstrate an understanding of focal points in a conic section.
- AM.7.3.** Analyze a conic section in terms of its eccentricity.



Additional Math Advanced Mathematic Topics:

(Outcome details determined by the teacher.)

- Fractal geometry
- Calculus topics (to extend beyond Introduction to Calculus content)
- History of mathematics
- Applications of mathematics to computer science (e.g., cryptography)
- Combinatorics extending beyond permutations and combinations (e.g., pigeonhole principle)
- Interdisciplinary project

Curriculum Implementation Resources

Curriculum implementation resources are frequently added. Please refer to https://www.edu.gov.mb.ca/k12/framework/english/math/resources/grade_12.html.