

# Grade 12 Pre-Calculus Mathematics

Course Code 3939

Course Credit 1.0

**ENGLISH** Program

## Discipline Overview

The Kindergarten to Grade 12 mathematics curriculum in the English Program is designed to meet learners' interests, skills, and needs so they can realize that mathematics is a way of building their understanding of the world and that it is part of their everyday lives.

The learning outcomes of this program of study are divided into four areas:

- Number
- Patterns and relations
- Shape and space
- Statistics (starting in Grade 2) and probability (starting in Grade 5)

These areas reflect the nature of mathematics from Kindergarten to Grade 12.

The study of mathematics promotes the development of global competencies and supports learners' cultivation of enduring understandings. It also fosters the development of logical thinking, problem-solving, and data analysis skills.

Learning experiences and evaluation in the mathematics classroom are based on a problem-based learning approach that allows learners to make connections between their conceptual understanding and various mathematical processes. Integrating these processes into learning helps learners to understand the nature of mathematics, and to make sense of it so they can learn and use mathematics in and out of school throughout their lives.

## Course Overview

Grade 12 Pre-Calculus Mathematics learners will develop trigonometric reasoning by demonstrating an understanding of angles in standard position, developing and applying the equation of the unit circle, and using the six trigonometric ratios for angles. They will represent graphically and analyze trigonometric functions, solve trigonometric equations algebraically and graphically, and prove trigonometric identities. Learners will develop algebraic and graphical reasoning through the study of relations by demonstrating an understanding of the effect

of different transformations on graphs of functions and their related equation. They will also demonstrate an understanding of inverses of relations, of logarithms, and of graphing and analyzing a variety of functions. They will develop algebraic and graphical reasoning that involves combinatorics by applying the counting principles, determining the number of permutations and combinations, as well as expanding the powers of a binomial in a variety of ways.

## Global Competencies in Mathematics



### Critical Thinking

**Critical thinking in mathematics** involves the ability to compare, evaluate, critique, justify, test, and validate ideas, representations, plans, or solutions using logical arguments, criteria, and evidence. It requires metacognition in learners, enabling them to solve mathematical problems and situations, communicate their reasoning effectively, and make ethical decisions.

- Learners research, use, and think about a variety of ideas and information strategically, efficiently, and effectively to make decisions and choices.
- Learners evaluate their own and others' ideas, as well as possible solutions, by considering different perspectives, biases, and the validity and relevance of supporting sources.
- Learners use inductive reasoning to explore and record results; to analyze mathematical ideas, problems, and situations; to make observations and generalizations from patterns; and to test these generalizations based on criteria and evidence.
- Learners recognize that certain math beliefs influence how they perceive themselves as math learners.
- Learners demonstrate a willingness to reconsider their own thinking and to consider others' thinking about mathematical ideas, problems, or situations.
- Learners ask relevant and clarifying questions to further learning and enhance comprehension of mathematical ideas, concepts, problems, and situations.
- Learners make judgments based on thoughtful criteria to then make decisions and solve mathematical problems and situations, enabling them to take action in an informed manner.
- Learners use deductive reasoning to solve mathematical problems and situations, reach new conclusions based on what is already known or assumed to be true, and make ethical decisions.



## Creativity

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**Creativity in mathematics** involves flexible thinking, curiosity, and risk taking, as well as making connections to prior knowledge among learners; this allows learners to come up with innovative solutions to a variety of mathematical problems and situations by considering them from a new angle or by formulating new hypotheses.

- Learners embrace a learning environment of trust and respect that encourages them to make choices, take risks, and think flexibly—allowing them to make decisions and take action.
- Learners wonder, ask questions, and contemplate different mathematical ideas and concepts.
- Learners solve mathematical problems and situations using different ways to arrive at innovative solutions.
- Learners enrich and refine their reasoning by considering others' ideas.
- Learners formulate, adjust, and refine their plans for solving mathematical problems and situations by looking at them from a new angle.
- Learners validate and adapt plans, ideas, strategies, or solutions, while persevering through obstacles, so they can improve at solving mathematical problems and situations.
- Learners seek and use feedback from others to develop and consolidate their conceptual understanding, deepen their reasoning, and reflect on their processes for solving mathematical problems and situations.



## Citizenship

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**Citizenship in mathematics** involves the development of mathematical literacy that enables the application of mathematical ideas and concepts in a variety of everyday contexts, awakening learners' curiosity about their role as citizens who can actively contribute to society, think critically about the world, make informed decisions, and generate solutions to an issue from a variety of perspectives.

- Learners use mathematics as a means of developing their understanding of a range of complex social, cultural, economic, and political issues, and to help them reflect on them.
- Learners mobilize their mathematical knowledge and skills to analyze and understand issues related to discrimination, equity, and human rights by investigating or proposing solutions to a variety of mathematical problems or situations related to these issues.
- Learners mobilize their mathematical knowledge and skills to explore, analyze, and understand the impact of the interconnectedness of self, others, and the natural world by investigating or proposing solutions to a variety of mathematical problems and situations related to this issue.

- Learners show interest in others' approaches to mathematics and to different points of view, experiences, and worldviews, allowing them to better understand and solve mathematical problems and situations.
- Learners empathize with others whose ideas are different from their own and appreciate solutions to mathematical problems or situations proposed by others.
- Learners interact and learn with others in person or online in a responsible, respectful, and inclusive manner by welcoming and valuing diverse viewpoints, and by considering a range of ideas and perspectives when contributing to mathematical exchanges.
- Learners realize that their mathematical knowledge and skills will serve not only to improve their own quality of life but also that of others.
- Learners engage in meaningful mathematical inquiries, individually and in collaboration, in which they ask themselves and others questions so they can find equitable solutions and make ethical decisions.
- Learners appreciate how mathematics can be used to make and justify ethical decisions that lead to responsible and sustainable actions that affect themselves, their community, and the world.



## Connection to Self

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**Connection to self in mathematics** involves the learner's belief in their ability to approach and complete tasks, solve mathematical problems and situations, and persevere in the face of mathematical challenges. It also involves the learner's ability to engage positively in reflective practices about their learning in order to set goals for self-improvement.

- Learners believe in their ability to learn and understand the world of mathematics and its impact on their daily lives.
- Learners recognize the elements that shape their identity as math learners, and they see themselves as mathematicians.
- Learners allow themselves the time they need, and they implement strategies that foster a growth mindset to develop a positive relationship with mathematics.
- Learners consider reflecting on their own decisions, the efforts they deploy, the experiences they have, and feedback from others as learning opportunities to improve their knowledge and skills in mathematics.
- Learners reflect on their mathematical learning to set goals and make informed decisions that affect their well-being.

- Learners believe that their ability to learn, their talents, and their skills in mathematics will continue to improve throughout their lives through their hard work, perseverance, and effort.
- Learners are willing to take risks, ask for help, and persevere, despite obstacles.
- Learners demonstrate the ability to make changes and adapt to new mathematical contexts, knowing that they will learn from their mistakes and build on their personal strengths.
- Learners develop their autonomy, value their voice, and commit to their role in becoming lifelong mathematics learners.



## Collaboration

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**Collaboration in mathematics** involves adhering to a culture of exchanging ideas and viewpoints in order to improve, both collectively and individually, and to learn from and with others to develop and apply new ideas in mathematics.

- Learners collaborate with others, value diverse points of view, and consider a range of ideas and perspectives when contributing to mathematical exchanges.
- Learners participate actively and fully in learning experiences by sharing thinking and learning strategies with others to confirm or extend understandings of mathematical ideas; they respectfully voice their opinions, ideas, and conjectures.
- Learners value the contributions of others, making room for different points of view that will foster mathematical exchanges.
- Learners practise active listening, question their own and others' mathematical ways of thinking, and ask questions of others to deepen their understanding of mathematical concepts and ideas.
- Learners show a willingness to compromise and change their opinions when presented with convincing arguments during mathematical exchanges.
- Learners make sense of mathematical concepts and ideas by co-constructing their understanding with others.
- Learners support others and take responsibility for their roles throughout the learning process and in the execution of mathematical tasks.



## Communication

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**Communication in mathematics** involves the learners' ability to share their mathematical ideas, reasoning, and solutions in a variety of ways, including orally, in writing, concretely, graphically, and symbolically, and in various contexts. It enables learners to clarify and validate their ideas and reasoning, and to challenge their attitudes and beliefs about mathematics.

- Learners express their mathematical ideas and emotions about mathematics, taking into account non-verbal cues and adjusting what they say according to the context.
- Learners present their mathematical ideas visually, orally, in writing, graphically, or symbolically, taking into account the conventions related to the mode of communication used, their audience, and the types of communication contexts, while using clear, precise mathematical language.
- Learners understand how their words and actions shape their identity as mathematical learners and shape their relationships with others.
- Learners look for oral, non-verbal, or visual cues during exchanges to improve their understanding of terminology, what others are saying, ideas presented, and various solutions to mathematical problems and situations.
- Learners seek to understand different points of view and different solutions to a mathematical problem or situation by observing, practising active listening, and asking clarifying questions, thereby creating a culture of mutual communication.
- Learners recognize and accept that the ways they learn and represent their understanding may be different from those of others.
- Learners make sense of mathematical ideas, problems, and situations, and deepen their understanding, by making connections among their own language, mathematical terminology, and mathematical conventions.
- Learners contribute to mathematical exchanges and express their thoughts and emotions about mathematical ideas in a positive and respectful way, whether in person or online.
- Learners defend their points of view and their mathematical reasoning while accepting the points of view and reasoning of others in a constructive and responsible way; they understand how these exchanges benefit themselves as much as they do other members of their learning community.



## Enduring Understandings

### Mathematics, a tool for growth

Developing knowledge and skills in mathematics allows for the understanding of processes, theorems, concepts, situations, and their applications. Building mathematical knowledge allows learners to identify as citizens who believe in their ability to complete a task, a lesson, or a challenge successfully—motivating them to take action and persevere throughout their lifetime to achieve their goals.

### Mathematics, a tool for seeing things differently

Mathematics is a part of the living world. It develops one's capacity to think fluidly and creatively, and to understand, interpret, and logically represent various phenomena that can be imperceptible or abstract, to put these phenomena in perspective regarding one another, and to analyze them from a different point of view.

### Mathematics, a vital tool for understanding the world

Mathematics contributes to the analysis, the comprehension, the interpretation, and the description of the world in which we live. It allows one to study quantities, orders, spaces, numbers, and figures, and the links that exist among them.

### Mathematics, an essential interdisciplinary tool for advancing society

Mathematics plays an important role in people's quest for innovation and solutions for the various problems encountered in advancing society. It is used in other disciplines to understand, describe, and interpret the phenomena that surround us, and it allows researchers to improve quality of life through innovation and the development of new technology.

### Mathematics, an essential tool for all aspects of daily life

Mathematics is everywhere in society. Mathematical knowledge and skills are necessary for critically analyzing information in diverse social, personal, economic, political, cultural, and environmental contexts in order to make informed, ethical, and sustainable decisions in all aspects of daily life.



## Learning Outcomes

### Mathematical Processes

There are critical components that learners must encounter in a mathematics program in order to achieve the goals of mathematics education and encourage lifelong learning in mathematics.

Learners are expected to

- **communicate [C]** in order to learn and express their understanding
- **connect [CN]** mathematical ideas to other concepts in mathematics, to everyday experiences, and to other disciplines
- demonstrate fluency with **mental mathematics and estimation [ME]**

- develop and apply new mathematical knowledge through **problem solving [PS]**
- develop mathematical **reasoning [R]**
- select and use **technologies [T]** as tools for learning and solving problems
- develop **visualization [V]** skills to assist in processing information, making connections, and solving problems

The *Common Curriculum Framework* incorporates these seven interrelated mathematical processes that are intended to permeate teaching and learning.


## Trigonometry

**General Learning Outcome:** Develop trigonometric reasoning.

- 12P.T.1.** Demonstrate an understanding of angles in standard position, expressed in degrees and radians.  
[C, CN, ME, R, V]
- 12P.T.2.** Develop and apply the equation of the unit circle.  
[CN, R, V]
- 12P.T.3.** Solve problems, using the six trigonometric ratios for angles expressed in radians and degrees.  
[C, ME, PS, R, T, V]
- 12P.T.4.** Graph and analyze the trigonometric functions sine, cosine, and tangent to solve problems.  
[C, CN, PS, T, V]
- 12P.T.5.** Solve, algebraically and graphically, first- and second-degree trigonometric equations with the domain expressed in degrees and radians.  
[C, CN, PS, R, T, V]

Learners should be able to solve first-degree sine, cosine, and tangent double-angle trigonometric equations.



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- 12P.T.6.** Prove trigonometric identities, using
- reciprocal identities
  - quotient identities
  - Pythagorean identities
  - sum or difference identities (restricted to sine, cosine, and tangent)
  - double-angle identities (restricted to sine, cosine, and tangent)
- [C, R, T, V]

## Relations and Functions

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**General Learning Outcome:** Develop algebraic and graphical reasoning through the study of relations.


- 12P.R.1.** Demonstrate an understanding of operations on, and compositions of, functions.  
[CN, R, T, V]
- 12P.R.2.** Demonstrate an understanding of the effects of horizontal and vertical translations on the graphs of functions and their related equations.  
[C, CN, R, V]
- 12P.R.3.** Demonstrate an understanding of the effects of horizontal and vertical compressions and stretches on the graphs of functions and their related equations.  
[C, CN, R, V]

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A compression by a factor of  $a$  is the same as a stretch by a factor of  $\frac{1}{a}$ .

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- 12P.R.4.** Apply translations, compressions, and stretches to the graphs and equations of functions.  
[C, CN, R, V]
- 12P.R.5.** Demonstrate an understanding of the effects of reflections on the graphs of functions and their related equations, including reflections through the
- $x$ -axis
  - $y$ -axis
  - line  $y = x$
- [C, CN, R, V]
- 12P.R.6.** Demonstrate an understanding of inverses of relations.  
[C, CN, R, V]

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- 12P.R.7.** Demonstrate an understanding of logarithms.  
[C, CN, ME, R]
- 12P.R.8.** Demonstrate an understanding of the product, quotient, and power laws of logarithms.  
[C, CN, R, T]
- 12P.R.9.** Graph and analyze exponential and logarithmic functions.  
[C, CN, T, V]
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It is intended that learners will be able to work with logarithms of any base,  $b > 1$ , including base  $e$ .

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- 12P.R.10.** Solve problems that involve exponential and logarithmic equations.  
[C, CN, PS, R]
- 12P.R.11.** Demonstrate an understanding of factoring polynomials of degree greater than 2 (limited to polynomials of degree  $\leq 5$  with integral coefficients).  
[C, CN, ME]
- 12P.R.12.** Graph and analyze polynomial functions (limited to polynomial functions of degree  $\leq 5$ ).  
[C, CN, PS, T, V]
- 12P.R.13.** Graph and analyze radical functions (limited to functions involving one radical).  
[C, CN, R, T, V]
- 12P.R.14.** Graph and analyze rational functions (limited to numerators and denominators that are monomials, binomials, or trinomials).  
[C, CN, R, T, V]

## Permutations, Combinations, and Binomial Theorem

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**General Learning Outcome:** Develop algebraic and numeric reasoning that involves combinatorics.

- 12P.P.1.** Apply the fundamental counting principle to solve problems.  
[C, CN, PS, R, V]



**12P.P.2.** Determine the number of permutations of  $n$  elements taken  $r$  at a time to solve problems.

[C, PS, R, V]

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Learners should be able to use strategies such as cases or grouping objects together to solve a contextual problem. Also, it is intended that circular permutations are not included.

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**12P.P.3.** Determine the number of combinations of  $n$  different elements taken  $r$  at a time to solve problems.

[C, PS, R, V]

**12P.P.4.** Expand powers of a binomial in a variety of ways, including using the binomial theorem (restricted to exponents that are natural numbers).

[C, CN, R, V]

## Curriculum Implementation Resources

Curriculum implementation resources are frequently added. Please refer to [https://www.edu.gov.mb.ca/k12/framework/english/math/resources/grade\\_12.html](https://www.edu.gov.mb.ca/k12/framework/english/math/resources/grade_12.html).